SOLAR DISTURBANCES AND TERRESTRIAL
TEMPERATURES

By W. J. HUMPHREYS

INTRODUCTION

The effort of this paper is to bring harmoniously together, in the sense of cause and effect, a number of solar and terrestrial phenomena, some of which are well known, while others, perhaps, are comparatively new and unfamiliar.

Since the sun is the source of practically all the radiant energy we receive, therefore any change in its surface or envelope that affects either the quantity or the quality of its radiation must, through the resulting modification of the energy received, also affect certain terrestrial phenomena, some of which are of vital importance.

Changes in the number and extent of its spots, flocculi, prominences, and coronal streamers, all constitute very obvious and probably closely connected disturbances of the sun; and many efforts have been made to determine what relation, if any, exists between these solar disturbances and such terrestrial phenomena as auroral displays, magnetic storms, temperature changes, rainfall, plant-growth, and, in fact, everything else that directly or indirectly can be connected with the sun.

That the magnitude and frequency of auroral displays and magnetic disturbances follow closely the corresponding changes in sun-spots appears to be well established. Concerning similar relations, however, of other phenomena to spot-frequency, the conclusions of investigators are less uniformly in accord, and perhaps only two at

most of these, namely, temperature and, possibly, rainfall, have been measured with sufficient accuracy and recorded long enough and at enough places to justify a minute examination of their relations to solar activity.

A careful study of the variation of rainfall in Europe with sun-spot numbers has been made by Hellmann,[1] who concludes that there is a slight increase in precipitation during sun-spot minima. This agrees with, and doubtless depends upon, the corresponding temperature changes; and as these latter are the most accurately measurable they alone, therefore, of meteorological phenomena, will be considered in what follows.

SUN-SPOTS AND EARTH TEMPERATURES

Of the various studies of the relation of terrestrial temperatures to sun-spot numbers, one of the latest and best is by Abbot and Fowle.[2]

The stations whose temperatures were used in this investigation were selected in accordance with the conclusions of a careful theoretical study of the problem to be solved, and the results obtained may, therefore, be accepted as the best available. From this paper it appears that:

1. Spot maxima are accompanied by temperature minima and spot minima by temperature maxima.

2. The average range, from spot maxima to spot minima, in the mean annual temperatures, at the selected stations, is about 1° C.

3. The decrease in radiation from the spot area itself can account for only about $\frac{1}{25}$ of the observed temperature change.

The above small range of 1° C. in the average yearly temperature, it will readily be admitted, is full of theoretical interest, but that it is also of great practical importance may not be so generally realized. However, this too will at once be granted when it is recalled that the growth of many plants, other things being equal, is, through a range of several degrees, roughly proportional to $N (T-T_0)$, in which N is the number of days of growth, T the average temperature during this time, and T_0 the critical temperature for the plant in question, or that temperature below which it will not grow.

[1] *Veröff. des Kön. Preuss. Meteorol. Instituts*, Berlin, 1909.

[2] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 177–201, 1908.

An excellent illustration of this general law, and one emphasizing the great importance to plant-growth of even small changes in the average temperature, has been given by Walter,[1] who shows that a change of only $0°7$ C. may alter, and in Mauritius has actually been observed to alter, by as much as an entire year, the time required for the development of sugar cane.

THE SOLAR ATMOSPHERE AS AFFECTED BY SUN-SPOTS

Surrounding sun-spots, and doubtless because of their accompanying or preceding eruptions, are extensive areas of flocculi, or great clouds of metallic vapors. Besides, the streamers and other phenomena of the corona, so conspicuous during a total eclipse of the sun, are much more pronounced and extensive during periods of spot maxima than at times of spot minima. It is also known, from observations made during total eclipses of the sun, that part at least of the coronal light is polarized, and therefore we infer that some of it is reflected or scattered light.

Sun-spot maxima, then, appear to be accompanied by maxima in the amounts of "dust" (any particles that perceptibly reflect or scatter light) and clouds in the solar atmosphere, which, because of diffuse scattering, and, possibly, selective absorption, may be expected to reduce the amount of radiation of short wave-length that is finally emitted.

SOLAR RADIATION AS AFFECTED BY THE SUN'S ATMOSPHERE

In this connection it is important to note the fact[2] that the solar intensity decreases from center to rim of the sun, that is to say, that the intensity decreases as the portion of the line of sight within the "dust" layer increases; and, especially, that this falling off of the intensity is much *greater* for the *shorter* than it is for the longer wave-lengths. The effective radiating layer of the sun, therefore, becomes cooler and, presumably, reaches higher levels as the rim is approached —results that would follow the phenomenon of diffuse scattering. In addition to this Abbot and Fowle[3] have shown that the distribution

[1] *On the Influence of Forests on Rainfall and the Probable Effect of "Déboisement" on Agriculture in Mauritius* (1908), 44–45.

[2] Hale and Adams, *Astrophysical Journal,* **25**, 300, 1907; Abbot and Fowle, *op. cit.,* Part 3, chap. i (1908); Adams, *Astrophysical Journal,* **31**, 30, 1910.

[3] *Astrophysical Journal,* **29**, 281, 1909.

of radiation in the solar spectrum can vary materially from time to time, especially in the region of short wave-length, even when there is but little if any change in the total intensity.

On examining Wolfer's[1] curves of sun-spot numbers it is seen that the days on which Abbot and Fowle found the amount of violet radiation to be relatively small were days of many spots, while the one on which they found it relatively large was a day of exceedingly few spots.

These observations, made on only four days in all, are not sufficiently numerous to justify, of themselves, any conclusions as to the relation of this change in the intensity of the violet to the size and number of sun-spots. Nevertheless, they are sufficient to show that changes in the violet do take place; and it seems highly probable that a decrease in the violet, through scattering, would be caused by the more extensive corona that prevails during the time of many spots.

It would appear reasonable, too, to expect some change in the total radiation output of the sun to accompany a change in the conditions of its surface, but the connection of any such change with the sun-spot period is not yet established, nor do we know enough about the causes of solar disturbances to give us confidence in theoretical deductions in regard to the total output of solar energy at times of spot maxima and minima.

Any change in the solar constant, other things being equal, would, of course, lead to corresponding changes in the surface temperatures of the earth; though, owing to the fact that the radiation of the earth is proportional, approximately, to the fourth power of its absolute temperature, and to the further fact that most of its surface is covered with water, the proportional change of the temperature would be much less than that of the total radiation, and, in point of time, would lag behind it.

A change in the relative intensity of the violet seems definitely established, both for different places on the sun and for the sun as a whole at different times, and it seems quite probable that this intensity must be least at the time of maximum sun-spot disturbance, or when the solar "dust" is most extensive.

It therefore is worth while to trace the effect on the temperature

[1] *Astronomische Mittheilung*, No. 98, 1907; No. 100, 1909.

of the earth of a change in the amount of violet and ultra-violet radiation, while the total energy output remains the same.

PRODUCTION OF OZONE BY ULTRA-VIOLET LIGHT

During the past few years a number of persons have examined and reported the production of ozone by the action of ultra-violet light on dry oxygen. One of the latest to investigate this subject carefully is Van Aubel,[1] who concludes that there can be no question about the ozonizing action of radiation of short wave-length. It is also known that this action is best when the gas is dry and very cold—the conditions that obtain in the upper portions of the earth's atmosphere.

One would, therefore, expect to find an appreciable amount of ozone in the upper atmosphere, where the conditions are the same as those under which it is best produced in the laboratory, but, owing to its unstable nature and its great power of oxidation, very little near the earth; and this is confirmed by direct observations. Only traces of ozone are found in the lower portions of the air, and when there is much moisture and the temperature is high the amount is vanishingly small. On the other hand, by examining the absorption bands in the solar spectrum, Ångström[2] has detected the presence of what seems to be a very considerable amount of ozone, which of course must be in the upper air, since it is not in the lower.

Now, ozone, as Ladenburg and Lehmann[3] have shown, has a number of absorption bands scattered over a wide range of the spectrum, from well within the ultra-violet to far out in the ultra-red. There is a strong band of wave-length $0.3\ \mu$ and less, a moderate one in the neighborhood of $0.6\ \mu$, another strong one at $4.7\ \mu$, and, besides several of lesser intensity at different places in the ultra-red, a very strong band extending from about $8.5\ \mu$ to approximately $10.5\ \mu$, and a small one at $11.4\ \mu$. The most intense portion of the solar spectrum falls about half-way between the first two of these bands, the one at $0.3\ \mu$ and the other at $0.6\ \mu$, while the great absorption band, from $8.5\ \mu$ to $10.5\ \mu$, and the small ones at $7.6\ \mu$ and $11.4\ \mu$ belong to the region of strong earth-radiation. Presumably there

[1] *Comptes Rendus,* **150,** 96, 1910.

[2] *Arkiv för Matematik, Astronomi och Fysik,* **1,** 395, 1904.

[3] *Annalen der Physik,* **21,** 305, 1906.

are other ozone bands of still greater wave-length, but the region to which they would belong does not appear to have been examined in this connection.

From a consideration of the positions and intensities of the absorption bands of ozone, though they are known to only $12.5\,\mu$, together
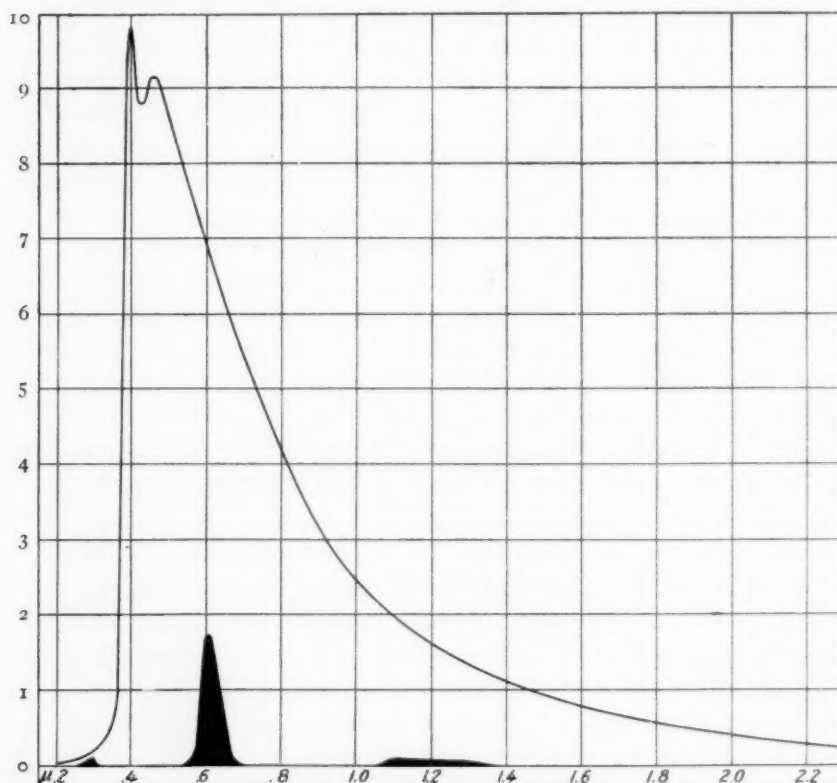
FIG. 1.—Ozone Absorption of Solar Radiation

with the distribution of energy in the solar and terrestrial spectra (assuming the earth to radiate as a black body), it is evident (1) that the coefficient of absorption, due to ozone, of solar radiation, is not the same as that of earth radiation, and (2) that the latter, probably, is greater, perhaps several fold greater, than the former.

These relations are graphically represented in Figs. 1 and 2, in which wave-length is plotted against intensity. Fig. 1 shows (a) the

approximate spectral distribution of the solar energy outside the earth's atmosphere,[1] and (*b*) the portion of this energy, indicated by the dark areas, that would be absorbed by a certain quantity of ozone.   Fig. 2 gives (*a*) the corresponding amount (only, because of loss by reflection, about two-thirds of the solar) and distribution of terrestrial radiation,[2] and (*b*) the fraction of this, in the region examined, that would be absorbed by the above assumed quantity of ozone.

These particular values of absorption were obtained by Ladenburg and Lehmann (*op. cit.*) with a tube of ozone one meter long, apparently
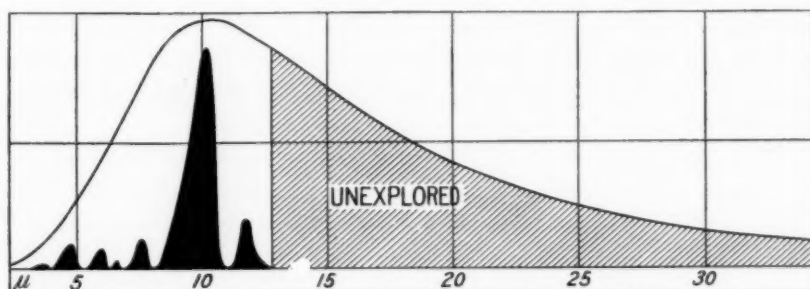
FIG. 2.—Ozone Absorption of Earth Radiation

at room temperatures and under a pressure of 200 to 300 mm.   Unfortunately they did not extend their examination beyond 12.5 μ, but still this is far enough to show that ozone absorbs earth radiation to a much greater extent than solar, or in other words, that it lets solar energy in more readily than it lets earth energy out.

Fig. 2, of course, does not accurately represent the average spectral distribution of the earth's planetary radiation, nor is this definitely known.   However, since water vapor—always present in the atmosphere—absorbs and radiates at ordinary temperatures very much as a black body, it probably does approach the actual distribution near enough for the present purpose, that is, to give some idea of the

[1] This curve is taken, with a slight modification, from Plate XVI, Vol. II, *Annals of the Astrophysical Observatory of the Smithsonian Institution* (Abbot and Fowle).   The ultra-violet is extended farther here than in the original curve, though without other justification than the fact that solar energy is known to extend beyond 0.3 μ.

[2] Really the radiation of a black body at the average temperature of the earth.

greater absorptive power ozone has for terrestrial than for solar radiation.

EFFECT OF A SELECTIVELY ABSORBING LAYER ON THE TEMPERATURE OF THE EARTH

The next step is to determine what effect a selectively absorbing layer, particularly an upper atmosphere rich in ozone, would have on the temperature of the earth.

The solution of this problem is complicated by the alternations between day and night; by the constant changes, during the day, in the sun's inclination; by reflection, especially from clouds, and doubtless by many other things that more or less modify the final result.   However, since the coefficient of absorption is independent, so far as we know, of intensity of radiation, it is possible, by the use of average values, to obtain an approximate solution.

The average intensity of the normal component of the solar radiation over the entire earth is the total radiation flux in a solar beam, at the earth's distance from the sun, whose cross-section is equal to the area of a terrestrial great circle, divided by the area of the earth's surface.

This average intensity then is $\dfrac{I_0 \pi R^2}{4\pi R^2} = \tfrac{1}{4} I_0$, in which R is the effective radius of the earth as an intercepter of solar energy, and I_0 the intensity of the solar radiation at the outer limit of the atmosphere, or the amount of solar radiant energy received per second at that place per unit area at right angles to the path of radiation.

A portion of this radiation is reflected by clouds and other things and of course produces no change in the temperature of the earth. If we adopt 37 per cent, the value determined by Abbot and Fowle,[1] as the total albedo of the earth, or the reflected fraction of all incident radiation, and represent by I'_0 the average effective intensity of the normal component of the solar radiation over the entire earth, then

$$I'_0 = 0.16 I_0, \text{ nearly.}$$

Let $aI'_0 =$ the amount of solar energy, both direct and reflected, absorbed by the outer atmosphere, per second per unit area. Approxi-

[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution,* **2**, 163, 1908.

mately, and for convenience we will assume exactly, half this is radiated to space and half to the earth, including in this term the lower atmosphere.

The earth, which we shall follow step by step as it would warm up from an initially cold condition, now receives $(1 - a) I'_0$ solar energy per unit time and unit area, and $\dfrac{a}{2} I'_0$ energy per same time and area from the absorbing outer layer, which, presumably, is roughly coincident with the isothermal layer and rich in ozone. These two together amount to $I'_0 \left(1 - \dfrac{a}{2} \right)$.

After a time the earth will send back an equally intense radiation of its own, but one in which the energy is very differently distributed as to wave-length. Therefore, when equilibrium with this radiation is established, $I'_0 \left(1 - \dfrac{a}{2} \right)$ is the intensity of the outgoing earth radiation also.

Let $b I'_0 \left(1 - \dfrac{a}{2} \right)$ be the part of this long-wave earth radiation absorbed by the outer layer. One-half of this, or $\dfrac{b}{2} I'_0 \left(1 - \dfrac{a}{2} \right)$, will be radiated back to the earth, there absorbed, and once more sent out. The next amount returned by the shell is $\left(\dfrac{b}{2} \right)^2 I'_0 \left(1 - \dfrac{a}{2} \right)$, and so on indefinitely.

The total intensity of radiation, therefore, reaching the earth, when completely warmed, is

$$I'_0 \left\{ 1 - \frac{a}{2} \right\} \ \left\{ 1 + \frac{b}{2} + \left(\frac{b}{2} \right)^2 + \ \ldots \ldots \ + \left(\frac{b}{2} \right)^\infty \right\}.$$

Now b is either unity, the maximum value possible, or less, probably much less, and positive. Therefore the terms inclosed by the second pair of braces form a rapidly converging series, whose maximum value, when $b = 1$, is 2, and whose minimum value, when $b = 0$, is 1, but whose actual value probably is not much greater than unity. Calling this sum S, we have $I'_0 \left(1 - \dfrac{a}{2} \right) S$ as the average intensity of the entire radiant energy used in heating the earth.

Since the albedo of the earth is roughly one-third, we can say that the sum of the incident and reflected energy passing through the outer layer of the atmosphere is approximately $\frac{4}{3}$ that originally incident, so that $\frac{1}{4}aI'_0$ represents the absorption per unit time and area of reflected radiation. If there was no absorption of solar radiation in the outer layer then, of the additional $\frac{3}{4}aI'_0$ that would go on per second to each unit area of the earth below, one-third, or $\frac{1}{4}aI'_0$, would be reflected. Therefore, if the outer absorbing layer did not exist, the average intensity of the energy used in heating the earth would be $I'_0\left(1-\dfrac{a}{2}\right)$, while that which actually does exist is $I'_0\left(1-\dfrac{a}{2}\right)S$.

Let $b=0.20$, a value, as shown by Fig. 2, possibly of the correct order; then $S=1.11$, or the radiant energy actually received by the earth is about 11 per cent greater than it would be if there was no absorptive layer.

The average temperature of the earth is $14^\circ.4$ C.[1] or $287^\circ.4$ C. absolute. Therefore, since the percentage change in temperature is approximately one-fourth the percentage change in radiation, it follows, if the assumed value of b is correct, that the surface of the earth averages from 7° C. to 8° C., or say 13° F. warmer than it would be without the presence of the absorptive shell.

The amount of this difference in temperature depends, as we have seen, on the value of b, and it in turn upon the quantity of ozone or other absorptive material in the upper atmosphere. But ozone, as above explained, is produced by the action of ultra-violet radiation on cold dry oxygen. The amount of ozone in the outer atmosphere must, therefore, vary with the amount of ultra-violet radiation sent out by the sun, and this, it appears, is a variable quantity.

When this ultra-violet solar radiation is at a minimum, presumably during a sun-spot maximum, the amount of ozone in the upper layers of the atmosphere will be a minimum, unless maintained by some other process—see below—and the temperature of the earth will be correspondingly low. On the other hand, with a maximum of ultra-violet radiation, presumably during a sun-spot minimum, when the solar atmosphere is clearest, there will be a maximum amount of ozone, so far as this method of producing it is concerned, and a com-

[1] Hann, *Lehrbuch der Meteorologie*, 115.

paratively high temperature at the surface of the earth, even if the total radiation remains unchanged, which is improbable.

Unfortunately the amount of change in ultra-violet radiation during spot cycles, and its relation thereto, if any, have not yet been determined, and hence it appears desirable at this point to sum up the foregoing and sharply distinguish between facts and assumptions.

The facts are:

1. The brightness of the sun drops off from center to limb.

2. This drop is greater the shorter the wave-length, and is due, almost certainly, to diffuse scattering.

3. The total amount of violet and ultra-violet in the solar radiation changes from time to time.

4. This change is greater than the change in the solar constant.

5. During sun-spot maxima the solar atmosphere is more extensive—dustier than it is during spot minima.

6. The temperature of the earth is greatest during sun-spot minima, and least during spot maxima.

7. Rainfall appears to follow the temperature curve, and therefore to be greatest during spot minima.

8. Ultra-violet light acting on cold dry oxygen converts some of it into ozone.

9. There appears to be a considerable amount of ozone in the atmosphere, which must be at great elevations since it is not found, except in traces, and could not long exist in the lower atmosphere.

10. Ozone is selectively absorptive both of solar and of terrestrial radiation, but more absorptive of the latter than of the former.

11. An upper atmosphere, rich in ozone, would influence the temperature of the earth—the temperature becoming higher, other things being equal, the greater the quantity of ozone.

The assumptions are:

a) That the solar constant is the same during sun-spot maxima that it is during spot minima.

This assumption probably is not correct, but the amount of the change, and even its sign, are unknown, and therefore it is assumed to have no cyclic change; the assumption being made for the purpose of seeing whether or not the known temperature changes can be

accounted for, in any part, without invoking corresponding differences in the total intensity of solar radiation.

b) That the solar radiation is relatively weak in the shorter wavelengths during spot maxima. This appears all but necessarily true.

The conclusion is:

With the solar constant and other things, except, of necessity, the spectral distribution of the energy, unchanged, any proportionate decrease in the ultra-violet radiation, such as we believe takes place at the time of sun-spot maxima, leads to a diminished production of ozone in the upper atmosphere, an increase in its diathermacy, and a cooling off of the earth and the lower atmosphere.

PROBABLE AURORAL EFFECTS

So far only that production of ozone in the atmosphere has been considered which is due to the action of ultra-violet solar radiation.

It is known, however, that ozone is abundantly produced by the action of silent electric discharges, like those seen in a Geissler tube, on cool dry oxygen. Almost certainly, therefore, auroral displays, except those that exist above the oxygen level, and all other electrical discharges in the atmosphere must be accompanied by a greater or less production of ozone.

Auroral streamers invariably parallel, or nearly so, the earth's magnetic lines of force, and therefore, if of equal electric discharge throughout their paths, the density of this discharge must rapidly increase as either magnetic pole is approached, or, in general, with increase of latitude. Owing to the concentration of the lines of force and their nearer approach to the vertical as the distance from a magnetic pole is decreased, together with the latitude increase of the winds, there may be many electric discharges of the Foucault nature, in the atmosphere of high latitudes, that do not extend to equatorial regions. However this may be, it remains that auroral displays are seldom seen in the tropics, while they are of frequent—almost nightly —occurrence in high latitudes.

We would, therefore, expect that portion of the ozone which is due to auroral discharges to be most concentrated in the higher latitudes: (1), because of the increased density here of the discharges; and (2) because that while often probably above the oxygen level in

equatorial regions (where of course no ozone can be formed) they come down, by following the lines of magnetic force, well within the oxygen, where the production of ozone is possible, as the magnetic poles are approached.

Another disturbing element in this distribution is the rather slow equator-to-pole drift of the upper atmosphere, indicated by sounding balloons, and by the spread to high latitudes of the dust thrown out by Krakatoa, which must tend to deplete the tropical regions of their supply of ozone and to concentrate it in those on either side.

These two actions then, the equator-to-pole circulation of the upper atmosphere, and the paralleling of magnetic lines of force by auroral discharges, might very well lead to a greater amount of ozone in the upper air of temperate and polar regions than in that of the tropics. A consequence of this would be a greater diathermacy of the upper atmosphere in equatorial regions than elsewhere, and therefore a comparatively low temperature and great altitude, at this place, of the isothermal layer; a condition that actually obtains, for the under surface of the isothermal layer is fully 5 kilometers higher and 15° C. colder over the equator than in middle latitudes.

Now a sun-spot maximum is accompanied by increased auroral activities, and, presumably, by a minimum of ultra-violet radiation, while a spot minimum on the contrary gives few auroral displays, but a maximum, probably, of radiation of short wave-length. Therefore the amount of ozone in the atmosphere, since it results from the action both of auroral discharges and of ultra-violet radiation that seem to increase and decrease oppositely, though not necessarily equally, tends to remain more or less fixed; and consequently it appears quite probable that the variations in the average temperatures of the earth through a spot cycle are less than they otherwise would be —that the increase of auroras with spot numbers tends to maintain equal average temperature conditions of the earth, by more or less completely offsetting the effects due to the simultaneous decrease in the ultra-violet radiation.

Finally, since changes in the quality of the solar radiation must be more effective within the tropics than elsewhere, while auroras are active almost entirely at higher latitudes, it follows that the temperature changes in the two regions must be unequal, possibly

at times even in opposite directions. An increase of temperature in the first, at the time of a spot minimum, may be accompanied by a smaller increase, or at places even a decrease, in the second; while a decrease in the first, to be expected at the time of a spot maximum, may, because of auroras, be accompanied by a less decrease, conceivably even an increase, in the second.

However, since radiation is constant while auroras are fleeting and fitful, it seems probable that changes in the spectral distribution of the former rather than in the frequency of the latter may be the principal cause, through the resulting variations in the amount of ozone in the atmosphere, of the observed periodic swings in the average of terrestrial surface temperatures.

CONCLUSIONS

1. An increase in sun-spots appears certainly to be accompanied by a decrease in terrestrial temperatures, at least in many places, fully twenty fold that which can be accounted for by the decrease in radiation from the spot areas alone.

2. It seems nearly certain that sun-spot maxima, whatever the value at such times of the solar constant, must lead to a decrease in the ultra-violet radiation that reaches the earth, and a corresponding decrease in the production, by this method, of ozone in the upper atmosphere.

3. The increase in the auroral discharges that accompany spot maxima tend to increase the amount of ozone, especially in the higher latitudes.

4. The change in the temperature of the earth, and all its train of consequences, from spot maximum to spot minimum, is not necessarily dependent upon a change in the solar constant. It may depend largely, if not wholly, upon a change in the absorptive property of the atmosphere, caused, we believe, by a variation in the amount of ozone produced by ultra-violet radiation and by auroral discharges.

SUGGESTIONS

In addition to a careful determination of the solar constant and terrestrial temperatures during one or more spot cycles, it would be well to measure, at the same time, the accompanying changes in the

ultra-violet portion of the radiation, and also to follow, over the same cycles, the temperature and height of the isothermal layer, and to note, if possible, the amount of ozone in the upper atmosphere.

The information here called for is difficult, though not impossible, to obtain; but much of it—it may be all—is essential, though perhaps not sufficient, to the solution of the complex problem concerning the relation of solar activities to terrestrial temperatures—a problem of great interest, both from the strictly scientific and from the purely utilitarian standpoints.

MOUNT WEATHER OBSERVATORY
BLUEMONT, VA.
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FORMULAE FOR THE SPECTRAL SERIES FOR THE ALKALI METALS AND HELIUM

By R. T. BIRGE

The recent extension of the principal series of the sodium spectrum to the forty-seventh member by R. W. Wood,[1] of the second subordinate series of sodium to the eleventh member by H. Zickendraht,[2] and of the principal series of potassium, rubidium, and caesium to the twenty-third, twenty-fourth, and twentieth members, respectively, by P. V. Bevan,[3] gives us a means of testing very accurately the various formulae proposed to represent spectral series. The author has found that the Kayser and Runge formula,[4] even with four undetermined coefficients, is quite inadequate. On the other hand, the Ritz[5] formula, with three undetermined coefficients and a universal constant, is quite adequate for substances with low atomic weights, but becomes less so, as we ascend from potassium to caesium, failing utterly in the case of the last-named substance.

In order to obtain the best possible results, all wave-lengths were first reduced to the new Fabry and Perot scale. This scale, founded on Michelson's[6] measurement of the red cadmium line, and remeasured by Benoit, Fabry, and Perot,[7] is undoubtedly more accurate than the Rowland values. The list of secondary standards, as determined by Fabry and Buisson,[8] extending from $\lambda=6495$ to $\lambda=2374$, gives us the material for a complete table of corrections, to be applied to the formerly accepted values. These corrections, when plotted, lie on a complicated curve, and cannot be wholly compensated for by adjustment of constants in the spectral formulae, especially when the actual wave-lengths are known very accurately, as in the case of helium.

[1] *Astrophysical Journal*, **29**, 97, 1909.

[2] *Annalen der Physik*, **31**, 233, 1910.

[3] *Phil. Mag.*, **19**, 195, 1910.

[4] Kayser's *Handbuch der Spectroscopie*, **2**, 514.

[5] *Annalen der Physik*, **12**, 264, 1903.

[6] Michelson and Benoit, *Mémoires du bureau internationales des poids et mesures*, **11**, 1, 1895.

[7] *Comptes Rendus*, **144**, 1082, 1907.   [8] *Ibid.*, **144**, 1155, 1907.

Therefore the following table of corrections, expressed in Å, has been prepared. From $\lambda = 2300$ to 4600, the comparison is between Fabry and Buisson's iron lines (terrestrial),[1] and Kayser's iron lines (terrestrial),[2] while between $\lambda = 4600$ and 6500, the comparison is between Fabry and Perot's solar lines,[3] and Rowland's solar lines.

CORRECTIONS FOR REDUCTION TO FABRY AND PEROT SCALE

λ in Å	Correction Å	λ in Å	Correction Å	λ in Å	Correction Å	λ in Å	Correction Å
2400	-0.08	3300	-0.12	4100	-0.15	5300	-0.16
2500	-0.08	3400	-0.13	4200	-0.16	5400	-0.18
2600	-0.08	3450	-0.14	4300	-0.16	5500	-0.20
2700	-0.09	3500	-0.15	4400	-0.17	5600	-0.21
2800	-0.10	3600	-0.15	4600	-0.17	5800	-0.21
2900	-0.11	3700	-0.15	4800	-0.17	6000	-0.21
3000	-0.11	3800	-0.15	5000	-0.17	6200	-0.20
3100	-0.11	3900	-0.14	5100	-0.16	6400	-0.21
3200	-0.11	4000	-0.15	5200	-0.15	6600	-0.22

The above corrections reduce wave-lengths at 20° C., 760 mm on the old scale, to wave-lengths at 15° C., 760 mm on the Fabry and Perot scale. To correct to *vacuo*, the Kayser and Runge[4] values of the index of refraction of air, at 15°, 760 mm, obtained by interpolation from their original curve, were used.

This reduction to the Fabry and Perot scale necessitates the calculation of a new universal constant (N_o) for use in the Ritz formula. The recent remeasurement of hydrogen δ and ϵ by Evershed,[5] together with the Rowland determinations of hydrogen a, β, and γ, gives us five accurately determined lines, each of which gives a value of N_o directly $\left[N_o = \dfrac{4 \times 10^8}{h} \right.$, where h is the constant of the hydrogen formula $\lambda = h \dfrac{m^2}{m^2 - 4} \Big]$. The values of these five hydrogen lines in air, and *in vacuo*, with the h derived from each, and the difference

[1] *Comptes Rendus*, **144**, 1155, 1907.

[2] *Annalen der Physik*, **3**, 195, 1900.

[3] Fabry and Perot, *Astrophysical Journal*, **15**, 261, 1902.

[4] H. Kayser, *op. cit.*, Bd. **1**, 719.

[5] *Astrophysical Journal*, **28**, 162, 1908.

between observed and calculated values, using the average h, are given below.

m	Letter	$\lambda_{(air)}$	$\lambda_{(vacuo)}$	h	$\lambda_{(obs.)} - \lambda_{(cal.)}$
3............	a	6563.045	6564.635	3647.019	-0.002 Å
4............	β	4861.527	4862.712	.035	$+0.018$
5............	γ	4340.634	4341.693	.022	$+0.001$
6............	δ	4101.900	4102.901	.023	$+0.001$
7............	ϵ	3970.212	3971.185	.007	-0.015
Average......				3647.021	

Then $N_o = \dfrac{4 \times 10^8}{h} = 109{,}678.6$.

The former Ritz value is 109,675.0. This new value gives, for hydrogen, exactly the same agreement between observed and calculated values as does the old.

In determining a formula to express the principal sodium series, it was immediately evident that the Kayser and Runge formula was unsatisfactory. In using their formula, Kayser and Runge worked with the wave-lengths in air, and Bevan,[1] in testing the complete sodium series by this formula, also used the air-values. Since the correction to be applied, in reducing to *vacuo*, varies from 1.63 Å to 0.73 Å, it might be thought that the error introduced by a failure so to reduce would be considerable. However, this was not found to be the case, and the conclusions reached by Bevan apply equally well, if the wave-lengths *in vacuo* are used, the differences between observed and calculated values reaching a maximum of 2.7 Å for the four-coefficient formula.

In calculating spectral series with any formula, certain lines are chosen for the determination of constants, as a least-square solution involves much labor. It is found, however, that the results vary considerably, according to the lines used, and the author, after much needless work, was led to the following general conclusions.

The lines for which n is large (near the head of the series) are fixed principally by the value of the convergence frequency (the fi st undetermined coefficient in all formulae). Small changes in the other constants do not affect these higher terms. The lower lines in the series, however, are greatly affected by changes in these other

[1] *Proc. Roy. Soc.*, A, **83**, 421, 1910.

constants. Therefore, in a formula with m undetermined coefficients, use one line near the head of the series, and the first $m-1$ lines of the series, to determine the m coefficients. This rule will ordinarily give results most nearly approximating a least-square solution. In determining what line near the head of the series can best be used, the differences between observed wave-lengths of successive lines are calculated, then the differences of these differences—these latter being plotted and a smooth curve drawn through them. Any line near the head of the series which corresponds to a point lying *on* this curve will give a good average result. Moreover, the divergence of the points from this curve gives the general experimental errors, which no formula will correct, and gives therefore the minimum difference between observed and calculated values, to be obtained by the use of the correct formula, assuming that such a one exists. The Ritz formula, for all the elements tried, save only caesium, gave differences in exact coincidence with this minimum difference.

The formula, as derived mathematically by Ritz, is an infinite series of unknown form. For the expansion Ritz used the semi-convergent development

$$v = A - \frac{N_0}{\left(n + a + \dfrac{b}{n^2} + \dfrac{c}{n^4} + \ldots\right)^2},$$

actually employing only the first three terms of the denominator. N_0 is the universal constant, A the convergence frequency of the series, and v the frequency $\left(\dfrac{10^8}{\lambda}\right)$ of term n ($n = 2, 3$, etc.). Ritz has given this formula in two different forms:

$$1. \quad v = A - \frac{N_0}{\left(n + a + \dfrac{b}{n^2}\right)^2}$$

$$2. \quad v = A - \frac{N_0}{\left(n + a + \dfrac{\beta}{N_0}[A - v]\right)^2}.$$

This latter form can be expanded, by successive approximations, into

$$v = A - \frac{N_0}{\left(n + a + \dfrac{\beta}{n^2} - \dfrac{2a\beta}{n^3} + \dfrac{3a^2\beta}{n^4} - \dfrac{4a^3\beta - 2\beta^2}{n^5} + \ldots\right)^2}.$$

It differs, however, from [1] only by infinitesimals of higher order. The first form has been used in the calculations, the results of which are given in the following tables. These tables also embody the differences between observed and calculated wave-lengths, recalculated with the published constants and *vacuo* frequencies of Ritz. The results vary considerably in some cases from those given in his papers, especially as regards the lower members of the series, and the discrepancy is only partly accounted for by the fact that the published constants are not given to a sufficient number of places.

The following tables give the results for the principal series (less refrangible member of the pairs) of the alkali metals and for the single-line principal series of helium.

TABLES

Column 1. Number of line in the series.
    2. Wave-length in air — Ångström units.
    3. Wave-length *in vacuo* (Fabry and Perot scale).
    4. Estimated maximum experimental error.
    5. Difference between observed and calculated wave-lengths, as published by Ritz.
    6. The above difference, as recalculated by the author with the same constants.
    7. The difference, computed with the new constants of the author, and with the Fabry and Perot wave-lengths of column 3.
    8. The observer of the line.
* denotes the lines used in determination of constants.

TABLE I

LITHIUM

New constants $N_0 = 109,678.6$ (universal)
$A = 43,482.115$
$a = -0.047423$
$b = +0.025801$

1 n	2 $\lambda_{(air)}$	3 $\lambda_{(vacuo)}$	4 E	5 R (old)	6 R_2	7 R'(new)	8 Observer
2......	6708.2	6709.83	0.2	±0.00*	+0.76	±0.00*	Kayser and Runge
3......	3232.77	3233.59	0.03	±0.00*	+0.05	±0.00*	" "
4......	2741.39	2742.10	0.03	+0.07	+0.06	−0.07	" "
5......	2562.60	2563.27	0.03	±0.00*	−0.09	−0.01	" "
6......	2475.13	2475.78	0.1	+ .20	−0.11	−0.23	" "
7......	2425.55	2426.19	0.1	+0.01	−0.08	−0.19	" "
8......	2394.54	2395.17	0.2	+0.03	−0.03	−0.15	" "
9......	2373.9	2374.52	−	+0.10	+0.14	±0.00*	Liveing and Dewar
10......	2359.4	2360.02	−	+0.22	+0.27	+0.15	" "

TABLE II

SODIUM

New constants $A = 41,450.083$
$a = +0.144335$
$b = -0.1130286$

1 n	2 $\lambda_{(air)}$	3 $\lambda_{(vacuo)}$	4 E	5 $R_{(old)}$	6 R_s	7 $R'_{(new)}$	8 Observer
2	[5896.16]	5897.563	0.00	±0.00*	+0.12	±0.00*	
3	3303.07	3303.895	0.03	+0.03	+0.04	±0.00*	Kayser and Runge
4	2852.91	2853.63	0.05	−0.10	−0.18	−0.04	" "
5	2680.46	2681.17	0.10	−0.08	−0.07	+0.09	" "
6	2593.98	2594.67	0.10	−0.13	−0.11	+0.09	" "
7	2543.82	2544.49		−0.22	−0.22	−0.05	Wood
8	2512.15	2512.90		−0.15	−0.15	+0.07	"
9	2490.70	2491.36				−0.04	"
10	75.60	76.26				+0.03	"
11	64.53	65.18				+0.10	"
12	56.02	56.67				+0.05	"
13	49.46	50.11				+0.04	"
14	44.24	44.89				−0.03	"
15	40.06	40.71				+0.01	"
16	36.70	37.35				+0.06	"
17	33.85	34.50				+0.04	"
18	31.43	32.08				−0.01	"
19	29.42	30.07				−0.02	"
20	27.72	28.37				−0.01	"
21	26.28	26.93				+0.02	"
22	25.00	25.65				+0.03	"
23	23.88	24.53				+0.01	"
24	22.90	23.55				±0.00	"
25	22.04	22.69				±0.00*	"
26	21.29	21.93				+0.02	"
27	20.60	21.25				+0.01	"
28	20.02	20.67				+0.04	"
29	19.50	20.15				+0.07	"
30	19.00	19.65				+0.06	"
31	18.44	19.09				−0.05	"
32	18.09	18.74				±0.00	"
33	17.71	18.36				−0.01	"
34	17.38	18.03				+0.01	"
35	17.10	17.75				+0.02	"
36	16.80	17.45				±0.00	"
37	16.56	17.21				+0.03	"
38	16.33	16.98				+0.04	"
39	16.11	16.76				+0.03	"
40	15.89	16.54				+0.02	"
41	15.70	16.35				+0.03	"
42	15.52	16.17				+0.02	"
43	15.37	16.02				+0.03	"
44	15.21	15.86				+0.04	"
45	15.06	15.71				+0.03	"
46	14.94	15.59				+0.05	"
47	14.78	15.43				+0.01	"
48	14.64	15.29				−0.01	"
49	14.50	15.15				−0.04	"

Experimental data seem to indicate, in the case of the alkali metals especially, a relation between the principal series and the

TABLE III
POTASSIUM

New constants $A = 35,006.51$
$a = +0.286917$
$b = -0.22083$

1 n	2 $\lambda_{(air)}$	3 $\lambda_{(vacuo)}$	4 E	5 $R_{(old)}$	6 R_2	7 $R'_{(new)}$	8 Observer
2.......	7699.30	7701.17	0.2	±0.00*		±0.00*	Average of all ob- servations
3.......	4047.36	4048.36	0.03	±0.00*		+0.01*	Kayser and Runge
4.......	3447.49	3448.34	0.03	+0.13		+0.04	" "
5.......	3217.76	3218.58	0.03	±0.00*		−0.02	" "
6.......	3102.37	3103.15	0.10	+0.10		+0.08	" "
7.......	3034.94	3035.71	0.10	−0.20		−0.26	" "
8.......	2992.33	2993.09	0.15	−0.19		−0.19	" "
9.......	63.36	64.11	0.20	−0.13		−0.25	" "
10.......	42.8	43.54	1.00	−0.27		−0.27	" "
11.......	28.0	28.74				+0.07	Bevan
12.......	16.6	17.33				+0.16	"
13.......	07.6	08.33				+0.09	"
14.......	00.4	01.13				−0.02	"
15.......	2894.6	2895.33				−0.10	"
16.......	89.7	90.43				−0.32	"
17.......	85.9	86.63				−0.25	"
18.......	82.9	83.63				±0.00*	"
19.......	80.3	81.02				+0.15	"
20.......	77.9	78.62				+0.09	"
21.......	75.8	76.52				+0.01	"
22.......	74.1	74.82				+0.07	"
23.......	72.5	73.22				±0.00	"
24.......	71.1	71.82				−0.04	"
25.......	70.0	70.72				+0.03	"

second subordinate (sharp) series, and both Rydberg[1] and Ritz have incorporated this connection in their formulae, Ritz writing his

$$\pm v = N_o \left\{ \frac{1}{\left(n + a + \dfrac{b}{n^2}\right)^2} - \frac{1}{\left(m + a' + \dfrac{b'}{m^2}\right)^2} \right\}$$

$m = 1.5$ $n = 2, 3,$ etc. for principal series.
$n = 2$ $m = 1.5, 2.5,$ etc. for second subordinate series.

It is evident that the first term of each series is the same, except that v is negative in one case, and positive in the other. In Ritz's formula the sign of v has no significance, so that the two lines are really identical.

[1] K. Svenska Vetensk. Akad. Hand., **23**, No. 11, 1890.

By comparison with the sodium formula already evaluated,

$$A \equiv \frac{N_o}{\left(1.5 + a' + \dfrac{b'}{(1.5)^2}\right)^2} = 41450.083$$

and, for the second subordinate series, using the derived values of N_o, a, and b,

$$\pm \nu = 109{,}678.6 \left\{ \frac{1}{\left(2 + .144335 - \dfrac{.1130286}{2^2}\right)^2} - \frac{1}{\left(m + a' + \dfrac{b'}{m^2}\right)^2} \right\}.$$

Thus, of the three normally arbitrary conditions arising from three undetermined coefficients, only one remains arbitrary in the case

TABLE IV

RUBIDIUM

New constants $A = 33{,}689.07$
$a = +0.345595$
$b = -0.265769$

n	$\lambda_{(vacuo)}$	E	$R_{(old)}$	$R'_{(new)}$	Observer
2	7952.38	?	±0.00*	±0.00*	H. Lehmann
3	4216.75	0.03	±0.00*	±0.00*	Kayser and Runge
4	3592.62	0.05	±0.00*	+0.20	" "
5	3351.87	0.05	−0.11	+0.16	" "
6	3230.08			−0.01	Ramage
7	3159.00			−0.22	Bevan
8	3113.79			−0.23	"
9	3082.68			−0.65	"
10	61.08			−0.41	"
11	44.97			−0.40	"
12	32.87			−0.26	"
13	23.37			−0.23	"
14	15.86			−0.18	"
15	09.86			−0.08	"
16	04.66			−0.27	"
17	00.76			−0.03	"
18	2997.06			−0.26	"
19	94.16			−0.21	"
20	91.76			−0.09	"
21	89.56			−0.13	"
22	87.66			−0.14	"
23	86.16			±0.00	"
24	84.86			+0.14	"
25	83.85			+0.41	"
26	82.85			+0.53	"

of the second subordinate series.   For the principal series' constants furnish us the value of A, and one relation between a' and b', namely,

TABLE V
CAESIUM

New constants $A = 31,394.23$
$a = +0.411966$
$b = -0.33269$

1 n	2 $\lambda_{(air)}$	4 E	5 $R_{(old)}$	7 $R'_{(new)}$	8 Observer
2...........	8949.92	?	±0.00*	−0.02*	H. Lehmann
3...........	4593.34	0.05	±0.00*	±0.00*	Kayser and Runge
4...........	3888.83	0.1	±0.00*	−0.32	" "
5...........	3617.08	0.3	−0.72	−1.12	" "
6...........	3477.25			−4.00	" "
7...........	3398.40			−2.80	Ramage
8...........	48.72			−1.37	"
9...........	13.5				Bevan
10...........	3288.9			−1.73	"
11...........	70.6			−1.72	"
12...........	57.1				"
13...........	45.9				"
14...........	37.6				"
15...........	30.9			−1.19	"
16...........	26.0				"
17...........	21.6				"
18...........	17.4			−0.27	"
19...........	14.3			±0.00*	"
20...........	11.8				"
21...........	09.6				"
22...........	08.0			+1.18	"

TABLE VI
HELIUM

New constants $A = 32,033.20$
$a = +0.011131$
$b = -0.004767$

1 n	2 $\lambda_{(air)}$	3 $\lambda_{(vacuo)}$	4 E	5 $R_{(old)}$	6 R_2	7 $R'_{(new)}$	8 Observer
2 ...	20,400		?	−80.		−20.	F. Paschen
3 ...	5015.732	5016.966	0.02	±0.000*	+0.010	±0.00*	Runge and Paschen
4 ...	3964.875	3965.848	0.02	±0.000*		±0.00*	" "
5 ...	3613.785	3614.662	0.02	±0.000*	+0.003	+0.01	" "
6 ...	3447.734	3448.575	0.02	−0.001		+0.025	" "
7 ...	3354.667	3355.499	0.02	−0.038		+0.01	" "
8 ...	3296.900 ·	3297.726	0.02	−0.032	−0.028	+0.03	" "
9 ...	3258.336	3259.152	0.02	−0.116		−0.04	" "
10 ...	3231.327	3232.139	0.02	−0.13	−0.11	−0.04	" "
11 ...	3211.626	3212.437	0.02	−0.12	−0.11	−0.04	" "
12 ...	3196.81	3197.62	0.02	−0.09	−0.10	−0.03	" "
14 ...	3176.6(?)			[+0.08]			" "

that relation whic'i will give a correct calculated value for the line $m = 1.5$ (or $n = 2$), which is common to both series. The infra-red line $\lambda = 11,381.1$ ($m = 2.5$) has been measured by Lewis with great accuracy. Therefore this line must be taken as the other determining one. Th: resulting coefficients are:

$$A = 24,493.93$$
$$a' = +0.151022$$
$$b' = -0.054833.$$

In the following table, column a gives the differences between observed and calculated values, using these constants.

TABLE VII

SODIUM, SECOND SUBORDINATE SERIES

1 m	2 $\lambda_{(air)}$	3 $\lambda_{(vacuo)}$	4 E	a	b	8 Observer
1.5....	−[5896.16]	− 5897.563		±0.00*	[−20.]	
2.5....	11,381.1	11,384.2	0.2	−0.16*	−0.04*	P. Lewis
3.5....	6154.62	6156.12	0.1	+0.67	+0.02*	Kayser and Runge
4.5....	5149.19	5150.47	0.1	+0.56	+0.19	" "
5.5....	4748.36	4749.52	0.15	+0.45	+0.28	" "
6.5....	4541.70	4542.80	0.20	+0.08	−0.03	Zickendraht
7.5....	4419.9	4420.96		+0.02	±0.00	"
8.5....	4341.4	4342.46		+0.20	+0.15	"
9.5....	4287.7	4288.75		−0.17	−0.17	"
10.5....	4249.5	4250.54		+0.11	±0.00*	"
11.5....	4220.5	4221.53		−0.41	−0.43	"
12.5....	4198.5	4199.52		−0.65	−0.75	"

Zickendraht estimates his experimental error as 0.05 Å at 4500, increasing to 0.3 Å at 4200. According to this, the differences are much greater than the experimental errors. But the wave-lengths, as determined by different observers, were measured under different conditions of excitation, and seem to vary considerably as a result. Thus for $m = 6.5$, Kayser gives $\lambda = 4542.75$ (error = 0.2 Å) while Zickendraht finds $\lambda = 4541.70$ (error = 0.05 Å). Taking these facts into consideration, the formula seems to be fairly satisfactory.

Deriving a formula for this series, independent of the principal series, so that $m = 1.5$ is not included, we get these new values for the constants

$$A = 24,493.41$$
$$a = +0.150440$$
$$b = -0.050864.$$

The differences between observed and calculated values, using these new values, are given in column b, of Table VII. They are seen to be a slight improvement, but if $m = 1.5$ had been included, there would have been absolutely no improvement. This is shown by the fact that the convergence frequency (A) calculated independently of the principal series ($A = 24,493.41$) differs from the A furnished by the principal series ($A = 24,493.93$) by only 5 units in the sixth place. Therefore the main relation between the principal series and second subordinate series (namely, that the difference between the convergence frequencies of these two series gives the frequency of the term common to the two series) is accurately true.

Now, as to the results for the principal series: The calculated differences, using the new constants, in the case of lithium, sodium, and potassium, lie practically within the estimated maximum errors given by Kayser and Runge. The experimental errors estimated by Wood and Bevan (.02 Å for sodium, o.2 Å for potassium, and o.3 to o.4 Å for rubidium) consider only the accidental variations in measurement, and do not take into account constant errors of any kind, such as would be caused by varying conditions of temperature and pressure, finite width of the lines (when caused by *assymmetrical* broadening), etc. A consideration of these statements, together with the fact that the calculated differences vary quite irregularly over the positive and negative region, is sufficient to show that the calculated differences must have been caused by experimental errors, and that, therefore, the Ritz formula is satisfactory.

For helium the estimated maximum error is o.02 Å, and here again the calculated differences have a maximum of twice this amount but it is instructive to note that, in all cases, the smaller the estimated error, the smaller are the maximum calculated differences, the two preserving an almost constant ratio. These conclusions do not hold at all in the case of caesium, where the differences have a maximum of 4 Å, with an estimated error of o.4 Å. This discrepancy may be considered also from the following point of view.

The Ritz formula has theoretically the form of an infinite series. How many terms of the series need actually be used depends directly upon the size of the coefficients of the several terms. But these coefficients are found to increase with increasing atomic weight, so

that higher terms of the theoretical series become less negligible, as the atomic weight increases.   This increase in coefficients is shown in the following table:

TABLE VIII

Substance	Atomic Weight	a	b
Hydrogen.............	1	0	0
Helium..............	4	0.0111	0.0047
Lithium	7	0.047	0.026
Sodium..............	23	0.144	0.113
Potassium...........	39	0.287	0.221
Rubidium...........	85	0.345	0.266
Caesium............	133	0.412	0.333

Moreover, in the case of Na, K, Rb, and Cs, the coefficients bear a definite relation to the atomic volumes, namely, that both a and b are directly proportional to the atomic volume.   Hicks,[1] using the formula

$$v = A - \frac{N_0}{\left(m + a + \dfrac{b}{m}\right)^2},$$

found similar relations.   These relations are shown below.

TABLE IX

Substance	a	b	b/a	Atomic Volume	$a/a.v.$	$b/a.v.$
Na.........	0.14433	−0.11302	0.784	23.606	613×10^{-5}	480×10^{-5}
K...........	0.28692	−0.22083	0.770	44.617	642 "	495 "
Rb..........	0.34559	−0.26577	0.768	56.05	627 "	473 "
Cs..........	0.41196	−0.33269	0.807	70.584	585 "	472 "

These results show far more than a chance coincidence, and indicate very clearly, as Hicks points out, that the coefficients have some relation to the physical properties of the atom.   These relationships are an argument for continuing the use of the Ritz formula, increasing the number of coefficients, if necessary, rather than seeking for an entirely new type of formula, as several authors have done, since Ritz first published his results.

[1] "A Critical Study of Spectral Series," *Proc. Roy. Soc.*, **83**, 226, 1910.

Conclusion.—We may conclude, then, that the Ritz formula, with three undetermined coefficients, will represent practically within experimental errors the principal series of helium, and of all the alkali metals, except caesium. Sodium is represented with especial accuracy, contrary to the conclusions of Bevan (*loc. cit.*). For sodium, also, the chief relation between the principal series and the second subordinate series is quite accurately fulfilled.

DEPARTMENT OF PHYSICS
UNIVERSITY OF WISCONSIN
June 1910

PLATE XII

1

2

3

4

5

1910
March 17
G.M.T.
5ʰ 26ᵐ

March 17
5ʰ 30ᵐ

March 18
4ʰ 47ᵐ

April 13
7ʰ 47ᵐ

April 28
4ʰ 22ᵐ
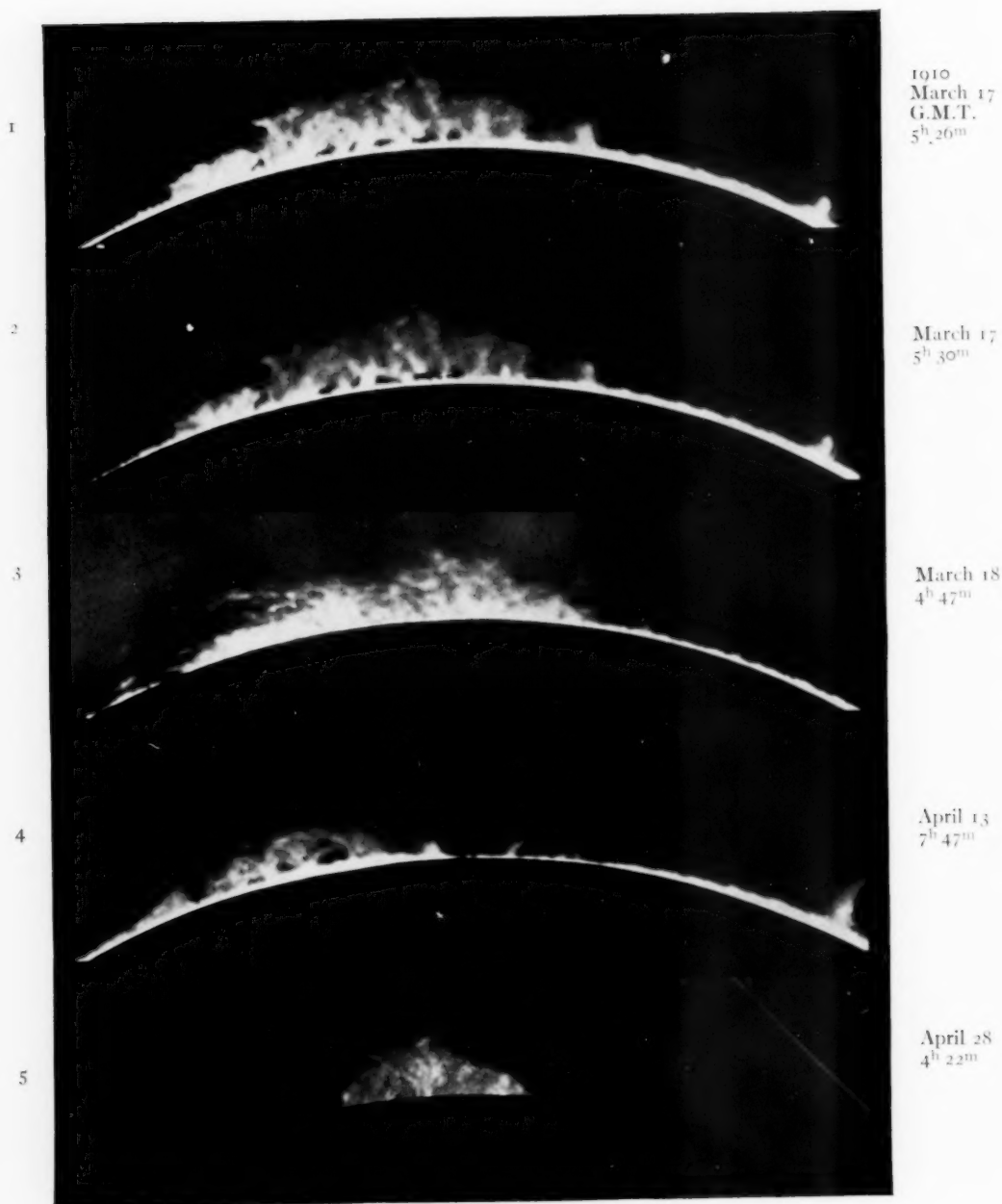
SPECTROHELIOGRAMS OF SOLAR PROMINENCES
Scale: Sun's Diameter = 225 mm

TWO SOLAR PROMINENCES

By FREDERICK SLOCUM

During March and April of the present year the surface of the sun presented an appearance characteristic of the approach to a minimum sun-spot period. Sun-spots were very few and small. Faculae and flocculi were weak and scattered. Solar prominences, on the other hand, were both numerous and active. Two prominences were of especial interest: one, on March 25, on account of its remarkable activity and brief existence; the other, observed from March 4 until April 28, on account of its great size, slow changes, and long life.

I. PROMINENCE OF MARCH 4—APRIL 28

The following table gives the chief facts in regard to the location of the prominence on the sun's limb and its dimensions on the various dates of observation. Plate XII shows reproductions of several of the photographs, all of which were made with the Rumford spectroheliograph, using the H_2 line of calcium.

Date	Number of Exposures	Limiting Position-Angles	Lateral Extent	Apparent Height	
1910			km		km
March 4	2	226°–249°	280,000	95.6	60,000
March 16	3	52°– 79°	328,000	84.9	61,000
March 17	6	48°– 83°	426,000	95.5	60,000
March 18	9	37°– 84°	572,000	106.1	77,000
March 31	2	224°–236°	146,000	29.6	21,000
April 1	1	222°–230°	97,000	16.1	12,000
April 12	2	63°– 71°	97,000	26.0	19,000
April 13	7	50°– 87°	450,000	64.3	47,000
April 14	3	59°– 80°	255,000	29.5	21,000
April 15	1	80°– 90°	122,000	24.1	18,000
April 27	1	225°–240°	182,000	67.1	49,000
April 28	5	218°–233°	182,000	83.1	61,000

When first seen on March 4 the prominence resembled a great compact mass with fine irregular markings on its surface, somewhat similar to those shown in the photograph of April 13, Fig. 4. It was then just passing around the western edge of the sun. On March 16 it reappeared, rising over the eastern edge, greater in extent

and transformed in appearance. By the next day it had reached its greatest apparent elevation, and resembled a grove of banyan trees with short, rugged trunks and intertwining branches. The general aspect remained the same throughout the day, but the minor details were constantly changing. Some such changes are shown in Figs. 1 and 2, Plate XII, which were taken on this date at an interval of 4 minutes. The most conspicuous difference is the bright jet which appeared near the right-hand edge of the main mass of the prominence at some time between $5^h\ 26^m$ and $5^h\ 30^m$. This particular feature may be an independent eruption projected against the prominence. Other differences may be noted in the size and shape of the tree trunks and in the position of the branches.

On March 18 the rotation of the sun had brought the prominence around so that the trunks and lower branches were projected against the disk and were therefore invisible. The tops of the branches remained visible and from these great streamers went out to the right and to the left, curving down to the photosphere, reaching over an arc of 47° of the sun's limb, or 572,000 km.

At this time the prominence extended from latitude −20° to about +25°, and its longitude was approximately 70°. This region was carefully examined on the calcium flocculi plates of the following days, but no trace of the prominence could be seen projected on the disk. Near the south end of the prominence was a scattered group of calcium flocculi, and about 20° following the north end was a small sun-spot closely surrounded by flocculi. Between these two there was nothing but the ordinary small granular flocculi which cover the whole disk of the sun.

No plates were obtained on March 29 or 30, but on March 31 and April 1 the top of the prominence was seen passing over the western edge. Twelve days later it reappeared once more on the eastern limb. Photographs were obtained on April 12, 13, 14, and 15. The maximum apparent elevation was reached on the afternoon of April 13. The appearance of the prominence at $7^h\ 47^m$ G.M.T. is shown in Fig. 4, Plate XII. Visual observations a few minutes later showed that the size and configuration of the prominence in the light of $H\alpha$, $H\beta$, and D_3 were essentially the same as shown on the photograph taken in the light of calcium H.

PLATE XIII

G.M.T. $4^h 14^m 7$

1

2

$4^h 57^m 0$

3

$5^h 45^m 0$

SPECTROHELIOGRAMS OF SOLAR PROMINENCES, MARCH 25, 1910

Scale: Sun's Diameter = 182 mm

During the following week the spectroheliograms of the disk were again carefully examined for traces of the prominence, but, as before, without success.

For the fifth time the prominence made its appearance on April 27 and 28 on the western edge of the sun. Its appearance had greatly changed. Its lateral extent was much less and its general appearance more like a compact mass, Fig. 5, Plate XII. Prominence plates were obtained on May 11, 12, 13, and 14, but no trace of a prominence could be seen where the great prominence had been. After an existence, probably continuous, of at least 55 days, it had at last vanished. Throughout the observations the southern extremity of the prominence remained in the vicinity of latitude $-20°$, while the northern limit varied greatly; starting near the equator on March 4, it moved up to $+25°$ on March 18 and then gradually retreated to $-10°$ on April 28.

II. PROMINENCE OF MARCH 25, 1910

This prominence was first noticed on a plate taken at $7^h 46^m.7$, G.M.T., March 24. It was conical in shape, the base extending from position-angle $230°$ to $235°$, and the apex rising to an apparent height of 46,500 km. The first plate of March 25, at $2^h 54^m.9$, showed that the form had become similar to a rugged oak tree 75,500 km high, the trunk at $229°.7$, and the branches extending from $225°$ to $235°$. From this time on, the development of the prominence was very rapid. At $4^h 14^m.7$ it had risen to a height of 120,000 km, Plate XIII, Fig. 1. This increase in elevation was produced in two ways: first, by the tapering aloft of the tree; and secondly, by the ascent of the tree as a whole, including the trunk and roots. No less than eleven rootlike filaments connected the trunk and spreading branches with the chromosphere. During the next forty minutes the prominence lost all resemblance to a tree. At $4^h 56^m.1$, Plate XIII, Fig. 2, two brilliant condensations at an altitude of 115,000 km were connected with the chromosphere by several filaments, and other fainter streamers reached up to a point 240,100 km high. The axis of the prominence, which had been normal to the surface of the sun, now appeared inclined $25°$ south of the vertical.

Between $4^h 56^m.1$ and $4^h 57^m.9$ occurred the maximum observed

motion, 11,600 km in 1.8 minutes, giving a rate of 107 km per second. At 5^h $10^m.6$ the two condensations had apparently blended and all the filaments united into one, giving a fair representation of a giant flamingo standing on one leg. At 5^h $43^m.0$ all that remained was a long thin filament rising, with a very slight curve toward the south, to a height of 281,000 km, then making a sharp turn and extending 97,000 km horizontally toward the south, and finally terminating in a cloud 74,000 km in diameter. The total length of the prominence

Date	G.M.T		No.	Plate Number	Height	
	h	m			"	km
1910 March 24...	7	46.7	1	3548	64	46,500
March 25...	2	54.9	2	3551	104	75,500
	4	14.7	3	3553	166	120,000
	4	16.7	4		171	123,900
	4	56.1	5	3554	332	240,100
	4	57.9	6		348	251,700
	5	10.6	7	3555	351	253,700
	5	43.0	8	3556	402	290,500*
	5	45.0	9		402	290,500
	5	55.4	10	3557	442	319,500*
	5	57.7	11		442	319,500
	7	43.3	12	3558	No	trace
	7	45.6	13			

* Exposure cut off below the highest point of the prominence.

as shown on this plate is 450,000 km, its apparent elevation 290,500 km. Fig. 3, Plate XIII, shows the prominence two minutes later. The point of the arrow marks the location of the faint terminal cloud, which shows clearly on the original negative. Unfortunately this and the following plates do not show quite the whole of the prominence, the exposure being cut off too near the sun's limb. At 5^h $55^m.4$ the filament had vanished, the detached cloud alone remaining, floating toward the south at a height of 319,500 km or $7'$ $22''$, and at a rate of 50 km per sec. On the next plate, at 7^h $43^m.3$, no trace of the prominence can be seen.

The dissolution of eruptive prominences by vanishing upward in this way, rather than by subsidence, has frequently been observed and seems to be a characteristic of this type of prominence.

The source of the prominence was in latitude $-16°$, longitude

144°. This region was carefully examined on the calcium plates of the disk taken on March 22, 23, 24, 25, but no trace of anything of especial interest was detected. The nearest disturbance on the disk was a large area of straggling flocculi from 5° to 15° north and from 5° to 25° east of the place of the prominence. The table on p. 128 gives a summary of the observations.

YERKES OBSERVATORY
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REMARKS ON WILSING AND SCHEINER'S MEMOIR ON THE TEMPERATURE OF 109 STARS

By J. WILSING

The review of the above memoir,[1] by Mr. Abbot, published in this *Journal* for April (**31**, 274, 1910) gives to the authors a welcome opportunity for making here a further statement as to the purpose and results of their investigation, particularly as the continuation of the measurements has furnished the material for settling one of the questions touched upon by Mr. Abbot.

The realization of Kirchhoff's black-body radiation renders it possible, in connection with the theoretical investigations by Wien and by Planck as to the form of Kirchhoff's function under certain conditions, to determine the temperature of a radiating body by measuring the energy-curve of the spectrum. These premises are thoroughly discussed in our Memoir, p. 20. In the first place, it is permissible to employ the radiation formula only in the case of pure temperature radiation. In the case of the temperature of celestial bodies, the question can in general be answered only a posteriori according to the agreement between measurement and theory. The latest investigations on the solar spectrum by Messrs. George E. Hale, Walter S. Adams, and others, in connection with laboratory experiments, permit us to regard the solar radiation as essentially a temperature radiation. Aside from the fulfilment of this condition, however, the transition from the values of the energy measured in the spectrum to the temperatures according to Kirchhoff's law requires the knowledge of the absorptive power of the radiating body. The object of the investigation could, therefore, be in the present case only the computation of the effective temperatures of the stars, i.e., we were to test whether the temperature of an absolutely black body can be so determined that the distribution of energy in its spectrum agrees with that in the solar spectrum within the errors of observation. Of course any value thus computed of the effective stellar tempera-

[1] J. Wilsing und J. Scheiner, "Temperatur Bestimmung von 109 helleren Sternen aus spectralphotometrischen Beobachtungen," *Publikationen des Astrophysikalischen Observatoriums zu Potsdam*, No. 56, **19**.

ture is to be regarded only as a mean value, since, as Mr. Abbot remarks and as was emphasized by the authors themselves (p. 43), the radiations of layers of the photosphere of different temperatures mingle, and the absorption, also, has an effect upon the distribution of the energy. In spite of this, in the opinion of the authors, the values of the temperature of the photosphere computed from the observed mean energy-curve may be compared with the value which could be computed from the temperatures of the separate radiating layers.

The comparison of the energy-curves of stellar spectra and of the black body (an electrically heated furnace of Heraeus, the temperature of which could be measured by a platinum, platinum-rhodium thermo-element) was made with the aid of an incandescent electric lamp with a carbon filament. From the ratio of intensity of the spectrum of furnace and of lamp, found by measurements with the spectral photometer, the energy in the spectrum of the lamp, E'_λ, was computed by means of the Planck formula for the radiation of the furnace

$$E_{o\lambda} = C\lambda^{-5}(e^{\frac{c}{\lambda T'}} - 1)^{-1}$$

in which T' denotes the absolute temperature of the furnace, λ the wave-length, c the constant of the law of radiation, C a second constant depending upon the particular conditions of the measurement. If $\log n_\lambda$ is the logarithm of the ratio of intensity between the spectrum of the star and of the lamp, photometrically determined, we obtain, computing the energy E_λ in the stellar spectrum and the temperature T of the star, equations of condition of the form

$$\log E_\lambda = \log n_\lambda + \log E'_\lambda + \Delta_\lambda = k - 5 \log \lambda - \frac{c \log e}{\lambda T} - \log(1 - e^{-\frac{c}{\lambda T}}).$$

Here Δ_λ denotes a correction covering the errors in the values assumed for the energy of the radiation of the lamp, for the selective absorption in the objective, for the coefficient of transmission of the atmosphere, and k is a constant. The last number on the right-hand side of the above equation is small, and can always be determined with an approximate value T_o, so that we have for the determination of the unknowns Δ_λ, x, y, or T, linear equations of the form

$$\log n_\lambda + \log E'_\lambda + 5 \log \lambda + \log (1 - e^{-\frac{c}{\lambda T_o}}) = x - \Delta_\lambda + \frac{\lambda_o}{\lambda}y. \qquad y = -\frac{c \log e}{\lambda_o T}.$$

The measurements were made at five places in the stellar spectrum as free as possible from lines, so that there are five equations of condition for the determination of the quantities x and y for each star, provided we may regard the corrections Δ_λ as insignificant. It appeared, however, that on this assumption the residuals from the adjustment of the measured values of the energy by the method of least squares had constant values for each color, independent of the temperature (see p. 43).

$$\log n \text{ (observed)} - \log n \text{ (computed)}$$

Temperature of Stars No. of Stars Weight	$3000° - 5000°$ 25 2	$5000° - 7000°$ 12 1	$7000° - 9000°$ 13 1	$9000° - 11000°$ 10 1	Mean
λ					
$0.448\,\mu$	+0.033	+0.039	+0.045	+0.037	+0.037
0.480	+0.005	+0.002	−0.005	+0.006	+0.003
0.513	−0.075	−0.068	−0.059	−0.064	−0.068
0.584	+0.025	+0.013	−0.014	−0.003	+0.008
0.638	+0.011	+0.022	+0.038	+0.036	+0.024

In order to disclose the cause of these deviations, measures were made in the spectrum of sunlight reflected from a chalk surface[1] and in the spectrum of the Argand lamp. The relations of these measurements, as well as of the energy-curves of the last two spectra of the incandescent lamp employed (which, similarly to those of the

[1] The following values were found for the logarithms of the coefficients of reflection Δ_λ (multiplied by a constant a) of the radiation reflected by the chalk (J. Wilsing and J. Scheiner, "Comparative Spectral Photometric Observations on the Moon and on Stones, etc.," *Publikationen des Astrophysikalischen Observatoriums*, No. 61, **20**, 16):

$$\log a\Delta_\lambda$$

λ	W.	S.	$\dfrac{W.+S.}{2}$
$0.451\,\mu$	9.992	9.992	9.992
0.472	0.020	0.010	0.015
0.494	0.001	9.991	9.996
0.514	9.997	0.017	0.007
0.535	9.993	9.985	9.989
0.556	9.999	0.005	0.002
0.577	9.983	0.014	9.999
0.593	0.007	9.992	0.000
0.615	9.990	0.014	0.002
0.642	0.022	9.985	0.004

The distribution of the energy in the spectrum of light diffusely reflected from the chalk is, therefore, according to these measurements, not different in the visual portion from that in the spectrum of the source itself.

Argand flame, should agree closely with the energy-curve of a black body) give the same residuals as were found in the case of stellar spectra. Thus on p. 48 the following summary will be found:

<center>log E (observed) $-$ log E (computed)</center>

λ	Argand Flame Temp. = 2050°	Incandescent Lamp, 1 1970°	Incandescent Lamp, 2 1850°	Sun 5200°	Star 3000°–11000°
0.448 μ	+0.020	+0.042	+0.030	+0.005	+0.033
0.480	+0.010	+0.012	+0.018	+0.028	+0.003
0.513	−0.046	−0.086	−0.074	−0.047	−0.067
0.584	−0.012	−0.003	−0.006	+0.005	+0.012
0.638	+0.027	+0.036	+0.032	+0.011	+0.019

It follows from this agreement that the distribution of energy in the spectra of all sources of light when referred to any desired fundamental spectrum, such as that of the sun, or of the Argand lamp, is completely represented by the Planck formula and that the numerical values of the differences

$$y_* - y_\odot$$

which are independent of the quantities Δ_λ, or $\dfrac{1}{T_*} - \dfrac{1}{T_\odot}$, are determined without qualification. But the assumption that

$$\Delta_\lambda = 0,$$

made in the adjustment of the observations, is not permissible, and in fact the determination of the values of the energy of the radiation of the lamp E' by means of the black body turns out to be erroneous by amounts notably in excess in some cases of the amounts to be expected from the agreement of the measures among themselves. As Mr. Abbot properly remarks, our determination of these corrections Δ_λ were not made from independent sources of measures and the residuals above given are therefore to be regarded only as the most probable values of the quantity $-\Delta_\lambda$. But according to the explanations given above, the uncertainty from this deficiency affects only the determination of the absolute temperature of the fundamental star from the comparison with the radiation of the black body. Thus, if, for instance, the temperature assumed for the sun required the correc-

tion $\Delta T_\odot$, the corresponding correction of the stellar temperature ΔT_* would be obtained directly from the equation

$$\Delta T_* = \left(\frac{T_*}{T_\odot}\right)^2 \Delta T_\odot .$$

Here the numbers found from the measurements were to be introduced for T_* and $T_\odot$. From the remarks at the close of this note, which are based upon new and independent measurements, it follows that the constant amount of error in the determination of the temperature, which may be ascribed to the differences between the most probable and the true values of the corrections Δ_λ, cannot for the hottest stars much exceed 1000°.

For comparison with his bolometric measurements on the solar spectrum, Mr. Abbot discussed our spectral-photometric observations on four different assumptions. Of these only the second is permissible. The values of log E assumed by the authors for the distribution of energy in the solar spectrum are given in our paper entitled "Vergleichende spectralphotometrische Beobachtungen am Monde . . . . ," p. 29. The following table contains our definitive values for log E and those computed by Mr. Abbot. The differences are so slight that in what follows Mr. Abbot's figures have been retained. The third column gives the energy of radiation when the intensity at $\lambda 0.448 \mu$ is placed at 1000. The fifth column gives the results of the bolometric measurements by Messrs. Abbot and Fowle.

λ	0.448μ	0.480μ	0.513μ	0.584μ	0.638μ
log E..................	9.928	0.014	0.033	0.018	0.006
log E (Abbot)..........	9.932	0.018	0.037	0.022	0.009
E $(c=14,600)$..........	1000	1219	1274	1230	1194
E $(c=14,200)$..........	1000	1167	1172	1052	975
Abbot and Fowle........	1000	1040	1000	893	800

In the attempt to explain the considerable difference between the bolometric and the spectral-photometric measurements, Mr. Abbot has overlooked the effect due to the uncertainty in the assumed numerical value of the radiation constant c. We pointed out on p. 41 of our Memoir that, according to determinations of

temperature made by Messrs. L. Holborn and S. Valentiner[1] with the nitrogen thermometer, the value $c = 14,600$ used here in the reductions is probably too large, and should be replaced by $c = 14,200$. We then obtained from the spectral-photometric measurements $T = 5500°$ for the solar temperature (see p. 41). The investigations recently completed at the Reichsanstalt in Berlin by S. Valentiner[2] seem to make certain the necessity for this correction. It also follows from these investigations that all the values of the tensions of the thermo-elements, platinum, platinum-rhodium, as previously extrapolated for all high temperatures, were too low. Hence a correction of $\Delta T_o = +7°$ (corresponding to a mean temperature of $1500°$ absolute) should be applied to the data of the thermo-element by which was marked the temperature of the furnace. After applying the corrections to the values of the energy of the radiation of the lamp for $c = 14,200$, and correcting the mean temperature of the black body ($1500°$ absolute) by $7°$, we obtain for the energy-curve of the sun the values in the next to the last row of Table I. If the energy in the solar spectrum at $\lambda 0.638\,\mu$ is taken at 100, we get the following figures:

$$\log E$$

	$\lambda 0.448$	$\lambda 0.480$	$\lambda 0.573$	$\lambda 0.584$	$\lambda 0.638$
A. and F.....	125	130	125	112	100
W. and S.....	103	120	120	108	100
Difference ...	+22	+10	+5	+4	0

Only at $\lambda 0.448\,\mu$ do the differences appreciably exceed the amount corresponding to the precision of the spectral-photometric measures, particularly since the differences still involve the uncertainty in the determination of the coefficient of transmission of the atmosphere. Further, photographic determinations of the coefficient of reflection of chalk, with which the writer is occupied at present, seem to yield a small decrease with the wave-length, which would still further reduce the difference between the measurements by the different observers by several per cent. Finally, it should be noted that between $\lambda 0.40\,\mu$ and $\lambda 0.45\,\mu$, and $\lambda 0.50\,\mu$ and $\lambda 0.54\,\mu$, according to the data of

[1] "Eine Vergleichung der optischen Temperaturskale mit dem Stickstoffthermometer bis 1600°," *Annalen der Physik*, **22**, 1, 1907.

[2] S. Valentiner, *op. cit.*, p. 309.

Messrs. Abbot and Fowle,[1] depressions of a solar nature exist which
may have affected the two sources of observations in a different
manner.

For the wave-length $\lambda_{max.}$ of the maximum of the energy, Mr.
Abbot finds from the bolometric measurements $\lambda = 0.460 \ \mu$ and
from the spectral photometric measurements $\lambda = 0.571 \ \mu$. The latter
value is based on the assumption that $c = 14,600$. From the formula
for the ratio of the solar radiation to the radiation of the black body
at wave-length λ

$$\frac{E_\odot}{E_o} = e^{-\frac{c}{\lambda}\left(\frac{1}{T_\odot} - \frac{1}{T_o}\right)},$$

we obtain as a correction $\Delta T_\odot$ of the solar temperature

$$\Delta T_\odot = \left(\frac{T_\odot}{T_o}\right)^2 \Delta T_o,$$

where ΔT_o is the change of the temperature of the black body.
Therefore for $\Delta T_o = +7°$, and $\dfrac{T_\odot}{T_o} = 3.7$,

$$\Delta T_\odot = +100°.$$

Hence, the solar temperature becomes $T_\odot = 5600°$, and the wave-
length of the maximum of intensity, according to Wien's law, is
$\lambda_{max.} = 0.523 \ \mu$, whereby the difference between the bolometric and
the spectral-photometric measurements is reduced almost by half.
A corresponding diminution occurs in the difference between the
value $T_\odot = 5962°$, derived by Messrs. Abbot and Fowle from their
pyrometric measures, and the temperature $5600°$ (formerly $5100°$),
computed from the spectral-photometric measurements with the
corrected constant of radiation. From Mr. Abbot's remark, "It
seems misleading to compute temperatures from a spectral range of
only $0.2 \ \mu$," it would seem that he has not considered the form of
the equations of condition from which the temperatures are deter-
mined. If λ_1 and λ_2 are the wave-lengths of the places in the spectrum
at which measurements of energy have been made, we have

$$\log \frac{E_1}{E_2} = -5 \log \frac{\lambda_1}{\lambda_2} - \frac{c \log e}{T}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right).$$

We obtain for the factor $\dfrac{1}{\lambda_1} - \dfrac{1}{\lambda_2}$ the value 0.7 (when we place $\lambda_1 =$

[1] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 108.

0.45 μ, and $\lambda_2 = 0.64\,\mu$). This factor becomes unity for measurements in the infra-red portion of the spectrum for $\lambda_1 = 0.7\,\mu$ and $\lambda_2 = 2.5\,\mu$, where the energy of the solar spectrum, according to the measures of Messrs. Abbot and Fowle, is but 1 per cent of its maximum value. The accuracy of the determination of temperature from the less refrangible portion of the spectrum is, therefore, with equal accuracy in the determination of the logarithmic ratio of intensity, only greater by one-half, in spite of the ninefold greater extent of spectrum than that obtained from the optical region of the spectrum covering only 0.2 μ. For deciding the question as to how far the distribution of energy in the stellar spectrum may be represented by the radiation formula holding good for the black body, it is indeed necessary to compare as large an extent as possible of both spectra. It is, therefore, very much to be regretted that measurements in the region of larger wave-lengths have only been made in the case of the sun.

After the completion of the series of observations published by the authors, the simple prism in the spectral-photometer was replaced by a Rutherford prism having a dispersion between $H\alpha$ and $H\gamma$ of 6.3. The energy-curve of the lamp was again determined, and at ten places, by numerous comparisons of the spectrum of the glow-lamp No. 2 with the spectrum of the black body.

$$\log E'$$

λ	W.	S.
0.451 μ	9.227	9.265
0.472	9.507	9.496
0.494	9.703	9.700
0.514	9.846	9.811
0.535	9.999	9.994
0.556	0.133	0.099
0.577	0.240	0.247
0.593	0.319	0.336
0.615	0.437	0.464
0.642	0.593	0.589

The spectrum of the Argand flame was then compared with that of the incandescent lamp, and the temperature of the flame was computed by Planck's formula from the measurements of each observer

with the values of log E' found by him.   The following table contains the differences between observation and computation:

DIFFERENCES BETWEEN OBSERVED AND COM-
PUTED VALUES OF

log E $(O.-C.)$

λ	W.	S.	Mean
0.451 μ	−0.039	−0.032	−0.036
0.472	+0.035	−0.004	+0.016
0.494	+0.016	+0.046	+0.013
0.514	−0.008	−0.012	−0.013
0.535	+0.001	+0.012	+0.007
0.556	−0.006	−0.019	−0.013
0.577	−0.011	−0.006	−0.009
0.593	+0.007	+0.006	+0.007
0.615	−0.028	+0.018	−0.005
0.642	+0.032	−0.004	+0.014
Number of observations	2	2	
Temperature	2050°	2050°	2050°

The residuals after the adjustment are represented within the uncertainty of the measurements, and the marked deviation in the green, which was previously observed, has disappeared entirely. This remark also applies to the measurements previously made in stellar spectra of the first type.   For comparison with the earlier measures, they have been reduced with the value $c = 14,600$, and with the uncorrected temperature indications of the thermo-element.

DIFFERENCES BETWEEN OBSERVED AND COMPUTED VALUES OF

log E $(O.-C.)$ Wilsing

λ	ζ *Pegasi*	α *Pegasi*	α *Leonis*	α *Lyrae*	γ *Geminorum*
0.451 μ	+0.010	−0.020	−0.043	−0.014	−0.026
0.472	+0.027	+0.082	+0.063	+0.010	+0.023
0.494	−0.015	+0.049	+0.052	−0.017	0.000
0.514	−0.017	−0.048	−0.036	−0.004	+0.027
0.535	−0.017	−0.068	−0.011	+0.010	+0.033
0.556	−0.001	−0.064	−0.033	+0.044	+0.008
0.577	−0.021	−0.015	−0.018	−0.006	−0.116
0.593	−0.008	−0.005	−0.028	+0.028	+0.019
0.615	−0.003	+0.025	−0.021	−0.029	+0.043
0.642	+0.041	+0.064	+0.072	−0.003	+0.027
Number of observations	4	2	4	2	2

DIFFERENCES BETWEEN OBSERVED AND COMPUTED VALUES OF

$\log E$ $(O. -C.)$ Scheiner

λ	ζ *Pegasi*	α *Pegasi*	α *Lyrae*	γ *Geminorum*
0.451 μ...............	−0.015	+0.039	−0.038	−0.048
0.472	−0.003	+0.026	−0.041	−0.052
0.494	+0.009	−0.036	−0.003	+0.027
0.514	−0.012	−0.033	+0.048	+0.050
0.535	+0.040	−0.017	+0.082	+0.082
0.556	+0.001	−0.052	+0.069	+0.046
0.577	+0.004	+0.030	−0.009	.0.012
0.593	−0.004	−0.013	−0.064	−0.021
0.615	−0.022	+0.009	−0.012	−0.061
0.642	+0.003	+0.043	−0.027	−0.010
Number of observations...	4	1	2	1
Weight		½		½

DIFFERENCES BETWEEN OBSERVED AND COMPUTED VALUES OF

$\log E$ $(O. -C.)$

λ	W.	MEAN	
		S.	$\frac{W.+S.}{2}$
0.451 μ.......	−0.018	−0.019	−0.019
0.472	+0.041	−0.019	+0.011
0.494	+0.014	+0.001	+0.008
0.574	−0.015	+0.015	0.000
0.535	−0.011	+0.052	+0.020
0.556	−0.009	+0.022	+0.007
0.577	−0.035	+0.001	−0.017
0.593	−0.010	−0.028	−0.019
0.615	+0.003	−0.020	−0.009
0.642	+0.040	−0.003	+0.019

The following summary contains the temperatures computed from the measures of each observer (with $c = 14,600$); the mean of the same, T (with $c = 14,600$, and $c = 14,200$); the effective temperatures T' as formerly computed (pp. 63 ff.); the mean temperatures of the spectral classes T'_M (p. 65 of our paper); and, finally, the differences $T-T'$ and $T-T'_M$.

The second series of measures appears, therefore, to yield a value for the temperature of the stars of the first type about 1300° too low. This difference notably exceeds the accidental uncertainty following from the adjustment of the errors. But we must take into account

an uncertainty of 0.01 to 0.02 in the logarithms of the separate values of the energy $\log E'_\lambda$ as computed from the internal agreement of the measurements; and that a difference of about 0.05 in the logarithms of the ratio of the energy of the positions of the spectrum situated at λ 0.448 and λ 0.638 is sufficient for explaining the difference of temperature in question. On the basis of the new measurements, we are therefore justified in the conclusion that the probable value of the corrections Δ_λ previously found differ only slightly from the true values. Furthermore, the difference between the temperatures derived from the two sources of measurements is diminished in proportion to the square of the temperature of the stars.

	W. $c=14600$	S. 14600	T		T' $c=14600$	T'_M $c=14600$	$T-T'$ $c=14600$	$T-T'_M$ $c=14600$
			$c=14600$	$c=14200$				
ζ *Pegasi*.........	7700°	8100°	7000°	9100°	9300°± 800°	9500°	−1400°	−1600°
α *Pegasi*.........	8900°	8500°	8700°	10500°	11500°±1100°	9600°	−2800°	− 900°
α *Leonis*.........	8700°		8700°	10500°	9400°± 800°	9500°	−700°	− 800°
α *Lyrae*..........	8200°	7900°	8100°	9500°		9600°		
γ *Geminorum*	6900°	6800°	6900°	8000°	10300°±1000°	8700°	−3400°	−1800°
					Mean......	......	−2100°	−1300°

In the opinion of the authors, this statement shows that the results of the earlier measures agree as well as could be expected, in view of the difficulty of such measurements, with those of the second series based upon the new determination of the constant of radiation of the lamp. The observations are now to be resumed, and will be extended to all stars down to magnitude 4, north of the equator.

POTSDAM
May 1910

STUDIES ON THE EMISSION OF GASES

II. ON SPECTRAL TUBES FOR USE WITH DIRECT CURRENT

By H. KONEN and W. JUNGJOHANN

1. Mention has often been made of the desirability of employing spectral tubes adapted for direct current. The advantages of Geissler tubes fed by an induction coil are very great on account of their convenience and their serviceability in the case of very slight quantities of gas; they fail, however, when it is desired to make quantitative investigations, such as measurements of the energy in gaseous spectra, or to obtain the greatest possible brilliancy. In the first case, the average intensities are comparatively small, even when electric vibrations having a large maximum current-strength are employed; and further, the complicated electrical conditions of the discharge, which vary from one apparatus to another in a manner very difficult to control, prevent in general a separation of the different constants of the gaseous spectra and the establishment of the conditions under which such spectra appear. Furthermore, anyone who has investigated spectra of Geissler tubes with great resolving power, such as that of a large Rowland grating, will have felt the desire to increase the intensity of his source of light. We may even assert that the inadequate knowledge of many gaseous spectra, particularly beyond the limit of the visible spectrum, is for the most part to be attributed to the relative faintness of the spectral tubes.

2. It is an obvious idea to employ the direct current instead of the induction coil. If this is done, we shall obtain electrical conditions which are accurately controllable, and the different portions of the circuit will differentiate themselves clearly and constantly. We cannot say, however, that the earlier attempts with direct current gave the success awaited. On account of the very considerable kathode drop in many gases, particularly at higher pressures, it is necessary to employ high-tension batteries for supplying the direct-current tubes. But if these have the ordinary form determined by the expense, space, and maintenance, it is possible to obtain from them

only a few milliamperes of current. Direct-current dynamos of corresponding capacity are not only very expensive, but are difficult to handle, and can hardly come into question for use in ordinary laboratory practice.

Even if the source of the current of sufficient tension and capacity is available, the work done at the kathode with large current is so great that very special precautions have to be taken in order to protect the tubes and electrodes from melting. So far as we see, these are the reasons why there are but few experiments recorded in this direction: the notably excellent success obtained by Paschen,[1] with quartz tubes using direct current from a battery of exceptionally large capacity and number of elements, was published when we had already carried out our experiments to a large extent, so that we have independently from him followed a similar procedure.

3. The properties of the Wehnelt kathode suggest the avoidance of the inconvenience of the high-potential battery by employing as kathode incandescent metallic surfaces covered with oxide. In fact, aside from Wehnelt himself, a number of other observers, as Wiedemann, Geiger, Janicki, have employed such tubes, and they have indeed been placed on the market by different firms, Some have used for source of light the gaseous discharge, while others have employed the arclike process of discharge at the anode, which occurs in consequence of the drop at the anode when the current-density is sufficiently increased by diminishing the size of the anode. This procedure is essentially similar to that recently described by A. Hagenbach[2] of an arc under diminished pressure. It is easy in using Wehnelt kathodes with a luminous potential to reach gaseous discharges having an intensity of several amperes, kathodes of sufficiently large surface being employed. In spectroscopic work, the point desired is density of current, rather than strength of current, which seems to have been overlooked by different persons, and since most gases absorb only slightly, it ultimately depends on the density of the strata. From this point of view the density of current hitherto attained cannot be designated as even large, inasmuch as tubes of large cross-section have been employed generally.

[1] *Annalen der Physik* (4), **27**, 532–570, 1908.
[2] *Physikalische Zeitschrift*, **10**, 649–657, 1909.

We have, ourselves, in part in common with Mr. J. Kyll, made a large number of experiments with spectral tubes having Wehnelt kathodes, in which we modified the shape very greatly. Thus we have finally reduced the diameter of the tube to less than one millimeter, and have attained current-density of over 60 amperes per sq. cm. The limit was by no means reached there, but it was practically impossible to employ glass tubes with such current-densities which were equivalent to heavy arc discharges. We have employed different forms of tubes which were inclosed in vessels of water, but we found it impossible to use currents of the magnitude mentioned for any length of time with glass tubes. Our experience with these tubes, with which measurements of intensity in the emission and absorption spectra were simultaneously carried out, will be described in detail elsewhere.

4. In spite of frequent successes, we have, however, finally abandoned the tubes above described having Wehnelt kathodes. The necessity of employing a special current for heating the kathodes, which must amount to 10 amperes for a larger surface of the kathode and produces much heat, calls for a rather large dimension of the tubes, which become inconvenient with their surrounding vessels. With such strong currents as we used, it was difficult to prevent the metal kathode from burning up occasionally despite every precaution. The disadvantages connected with this can only imperfectly be avoided by a previous equipment with a reserve of several incandescent kathodes, but above all, the glass vessels cannot withstand the necessary current-density. The tubes have to be made of considerable dimensions, inasmuch as the incandescent kathode, with the attachments necessary for its renewal, cannot be diminished beyond a certain limit. The use of quartz vessels is forbidden by the technical difficulties and the very large cost for institutions only moderately endowed.

5. We have now found that a simple modification of the tubes described furnishes a form adapted for many purposes which stands about midway between a tube planned for a high potential and a Wehnelt tube. We may, in fact, entirely do away with the heating current of the glowing kathode. In many gases it is sufficient with pressures of 10 mm and upward to use the induction impulse of a

small induction coil, or even merely to bring up a piece of ebonite, glass, or shellac, that has been rubbed, to a tube provided with an appropriate kathode, in order to produce a discharge, if only the potential available amounts to about 800 volts. It is then sufficient to attach to the kathode a small piece of platinum foil about 0.25 sq. cm in size, and 0.01 mm thickness, that has previously been covered with a thin sheet of oxide by dipping it in a solution of calcium nitrate + barium nitrate, and heating it in a Bunsen burner. The thinner this coat is, the more successfully will the tube operate, and the platinum sheet may be of much smaller size. If we bring up to such a tube, for instance, a stick of hard rubber that has been rubbed, we notice that with increase in potential a luminous preliminary discharge occurs which follows the arc discharge after a slight interval, for the most part almost directly. The arc discharge is characterized by the fact that a point of the platinum foil is raised to incandescence by the discharge itself. The fall of the kathode is thus reduced to a few volts and the discharge of the tubes sinks under some circumstances down to 100 volts and less. The use of energy is limited in effect to the positive column and the anode drop, so that the anode is most strongly heated, as in the case of the arc. The negative glow light is lacking.
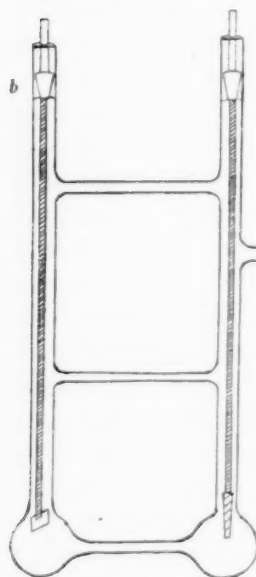
FIG. 1

6. After numerous preliminary experiments, we selected a tube of the form shown in Fig. 1. It was made by the firm of Heraeus from quartz glass, and can be connected with the pump by means of a ground attachment of glass. The length of the arms *bb* is about 22 cm. The spheres have a diameter of about 3 cm and the capillary is 60 mm long and 2 mm in internal width. It might be made narrower as well as longer. We have avoided this in order to be able to use the tubes also for experiments in absorption. At *b* are the

electrodes of nickel-steel with conical bearing, which can be made tight by pouring upon them a drop of mercury. The electrodes were made of steel wire of about 2 mm diameter, and they carry at their lower extremity a piece of steel wire several centimeters long. If necessary, this may be replaced by a piece of some other metal, such as aluminum. The end used as kathode carries a slit in which a small piece of platinum foil may be clamped.

7. As source of current, we used a dry battery of 420 elements, which in series gave a maximum current of 1.2 amperes or for a brief time 1.5 amperes. This was purchased with a grant from the Hittorf fund. In the hope that it might be enlarged in the future, it was very carefully set up and isolated by paraffin and thick plates of glass. Paraffin oil was poured over the acid in the ordinary way and the circuit leading to the battery was mounted on paraffin insulators. In spite of its relatively very small cost, this battery has served admirably without any interruption for several years. We have used it partly as a whole, and partly in parallel with 440 volts, in series, and partly also in groups of 140 elements arranged in series. We used the resistance bands made by Schniewindt of Neuenrade, i. W., which, in addition to their inexpensiveness, have the advantage of convenient mounting, slight self-induction, and high durability. The potential at the entrance of the tubes was measured with a standardized electroscope, as there was some difficulty in starting the tubes into action, when the measuring instruments did not take the current.

8. The tubes resembled very much the Wehnelt tubes. The discharge took place of itself at pressures which varied from gas to gas, and even at 840 volts. Where this was not the case, as already mentioned, an electrified rod of hard rubber was brought up. Its action was exhibited particularly at the anode. In many cases it was also sufficient to discharge the tubes by contact, or, better, by passing them through a flame. This procedure also proved to be necessary when the tubes had been running for a long time before the discharge could be produced by electrostatic action. The effect of the heat of a flame is felt in addition to that of conduction of the current; when the tube was heated to a faint redness by a Bunsen flame, the potential of discharge fell. It is natural to assume an action of ultra-violet light

here, since in these experiments an intense bluish-green phosphorescence of the quartz vessel was observed under the influence of the ultra-violet radiation of the Bunsen flame (the inner cone of which is particularly effective). This assumption, however, is not correct, as may be shown by check experiments with an arc lamp, and light concentrated upon a spark-gap by a quartz lens. The radiation from radium-bromide placed near by also failed to show an effect of reducing potential; but in our experiments, the radiation had first to penetrate 0.5 mm of glass and then the quartz glass, so that this may possibly explain the negative result. No effect was produced on bringing up a magnetic field, but we are still occupied with more accurate experiments on this point. Finally, it was also sufficient, as already mentioned, for starting the discharge, to allow a shock from a small induction coil to pass through the tube. For this, one tube was connected with the more remote electrode, the other with the platinum wire, which was set into the portion of glass connecting the tube with the pump.

9. It is not difficult with the means thus described to produce discharges at pressures of from 0.5 to 10 mm, hence just at the pressures which are involved in most spectroscopic work; the limits of pressure vary with the gas and with the tube. If it is desired to pass over to higher pressures, it is recommended that the discharge should be first introduced at lower pressure, and then to admit gas during the passage of the current. If exhaustion is carried below a definite limit, the glow discharge will first begin, then the current-strength falls, and finally the automatic passage of the current is extinguished. Within the range of pressure mentioned, the current-strength can be regulated in the simplest manner, however, and it remains constant to a fraction of a per cent. If, with tubes of the size described, the current-strength is appreciably less than 200 milliamperes, the glow discharge readily begins, the relatively small amount of heat produced at the kathode being conducted away and the remainder not being sufficient to produce the rise in temperature necessary to overcome the kathode drop. Under certain circumstances, we may then observe a periodic change from the arc to the glow discharge. After the glow discharge has lasted for a time, the temperature of the whole kathode rises in consequence of the kathode

drop, and the temperature of the platinum foil exceeds the limit necessary for the arc discharge. As soon as this happens and the arc discharge begins, the work done at the kathode decreases and the glow discharge begins again; and this process repeats itself. The glow discharge may be avoided by sufficiently reducing the oxidized surface and making it of thinner foil and connecting it with the steel kathode by a thinner wire. With an increase of current, the diameter of the incandescent spot at the kathode increases, but it is not until currents of o.7 amperes and upward are reached that signs are seen of an increased evaporation of the oxides covering the kathode. There is then formed directly upon the kathode an aureole of reddish-yellow color which shows, along with a few lines of calcium and barium, the oxide bands of the two metals. This remark is based solely upon visual observation.

In the spectra of the capillary in general nothing is noticed of the lines of the metals mentioned. It is advisable to avoid excess of oxide at the kathode, for in consequence of the evaporation of a portion of it there forms in the capillary a coating of oxide which attacks the quartz glass at incandescence and darkens it. This is of no significance for the ordinary use of the tube viewed lengthwise, inasmuch as the discoloration is limited to the capillary and does not occur until after weeks of use; furthermore, it may be entirely avoided with proper precautions. For this reason it is desirable always to observe from the side of the anode. With an extended use and the employment of strong current, the kathode is dissipated and gradually forms a brown coating on the inner side of the tube consisting of iron and platinum; but this is readily removed with a weak nitric acid.

10. We used currents as high as 1.5 amperes in the quartz tubes, but a further increase is prevented by the increasing heat of the anode, which glows vigorously at o.7 amperes. The capillary, too, is heated to a yellow glow at the current-strength mentioned. However, in consequence of the long arms of the tube, it is unnecessary to cool the electrodes.

With currents of from o.2 to 1.5 amperes, and corresponding current-densities of 6 to 30 amperes per sq. cm, the tubes are illuminated with different gases with a brilliancy similar to that of the arc. A strong odor of ozone is noticed in the vicinity, particularly with

CO in the tube, as in the case of mercury vapor lamps.   An exposure
of from one to two minutes was sufficient to obtain the whole spectrum
in the first and second order below λ 4800 on ordinary plates, with
a nitrogen tube used with a Rowland concave grating of 180 cm
focus and of a small ruled surface and moderate light-power.   A
concave grating of 4 meters radius gave a similar spectrum in 10
minutes in the third order.   Similar exposure times were found for
tubes with hydrogen, carbonic oxide, etc.   These numbers have, of
course, no absolute value, as they depend upon many variable con-
ditions; they suffice, however, to give a comparison with ordinary
spectral tubes with which exposures of hours are necessary when an
induction coil fed with a primary current of about 8 amperes and
having a striking distance of 50 cm is used.

11. The process of filling and cleaning the tubes does not differ
from that of ordinary tubes, and we refer here particularly to the paper
by Paschen, adding only a few remarks.   Disturbing impurities in
the tubes have been especially noticed in the case of hydrogen,
water-vapor, mercury, and carbon and its compounds.   The mercury
lines are particularly disturbing in photometric measures.   They
come in part from the mercury vapor from the pump, but are only
strong when the electrodes are not made very tight.   On putting
in the electrodes and pouring on the mercury the greatest care must
be taken that no trace of the metal enters into the tube.   In that case
the mercury vapor can be removed only by frequent heating, washing,
and complete exhaustion, and perhaps finally with the aid of liquid
air and carbon.   The hydrogen presumably comes from the electrodes,
which give off appreciable quantities of the gas for a long time.
The observer therefore generally obtains, in spite of careful drying,
the spectrum of water-vapor, doubtless in consequence of the pro-
duction of water by the oxide kathode simultaneously with the
reduction of the oxide by the hydrogen.   This production of hydrogen
diminishes after a time, however, and with it vanishes the hydrogen
spectrum.   This explanation of the appearance of the bands of water-
vapor is confirmed when the tubes are filled with hydrogen made
and dried with the greatest care.   We then obtained, along with the
lines of the series spectrum, the "second" hydrogen spectrum (many-
line spectrum) and at the same time lines of the strong bands of water-

vapor at λ 3064. The occurrence of the lines of hydrogen can further
be brought about by strongly heating the tube with a flame of hydrogen
or hydrocarbon; the hydrocarbon and hydrogen then diffuse into
the interior of the tubes.

The spectra of carbon and its compounds are due in part to
impurities in the tubes themselves, but to a greater extent to the carbon
of the steel electrodes. On filling the tube with nitrogen we therefore
obtained first the cyanogen spectrum and then the band spectrum
of carbon (Swan spectrum) developed to a greater or less extent, and
further the lines of carbon itself as impurities. If nitrogen is excluded,
then we often find, besides the spectra named, the third Deslandres
group of the so-called carbon-oxide bands present as an impurity.
All these impurities are disturbing at the start, but they disappear
with the passage of the current for a long time and with frequent
exhaustion, washing out with oxygen and with the gas used for filling
the tubes, which thus become very pure.

12. The precise determination of the electrical properties of the
tubes described presents decided difficulties. The measurements of
the fall of potential at the anode and kathode, as well as of the gradient
in the positive column, have not hitherto been made with the current-
densities like those used by us; they are also difficult because the
conducting wires are introduced with difficulty and melt off. If
we extrapolate from the measurements made with the strongest
currents hitherto employed, then we find a drop at the anode of about
15 to 20 volts on the assumption that in the region in question the
anode drop changes only slowly with the current-density, and that
the changes with the pressure, etc., may be neglected. The kathode
drop can be estimated from the measures made on the incandescent
oxidized kathode and it is in any case very small. We obtained
thus approximate values for the potential gradient, which at the same
time make it possible to estimate the consumption of energy per unit
of volume. The values so obtained agree for an unstratified dis-
charge approximately with those derived from previous measurements
on the dependence of the gradient in the positive column upon the
width of the tube, pressure, and current strength. A conclusion as
to these observations will follow later. They are mentioned here only
to indicate the possibility of an estimate of the consumption of energy

in the tube and the usefulness of more precise measurements of the anode drop. According to the strength of the current and the pressure, we obtain stratified or unstratified discharges. The observations which we have made with the telephone and a rotating mirror indicate that these are continuous within the precision of the method. By introducing appropriate capacity and self-induction we obtain oscillating discharges. In its general character the discharge stands about half-way between a glow discharge and an arc, and resembles in different respects the arc at diminished pressure, which Hagenbach (*loc. cit.*) recently described, and into which it may be converted by a continuous increase of the current-strength.

13. We have employed tubes of the sort described for different purposes with success; as a source in photographing the spectra of gases, it appears that band spectra in particular are furnished in great intensity. With these, using various line spectra, photometric measurements have been made, particularly by Jungjohann and by Kyll, on the relation of absorption and emission to pressure, current-strength, and consumption of energy; a report of these experiments will be made very soon, as well as upon their employment for experiments on the distribution of emission along the path of the discharge, etc. Under all of these circumstances the tubes we have described have proved satisfactory. It will be agreed that the sort of tubes thus described is suited to replace in many cases the ordinary vacuum tubes, when we reflect that they are useful not only for direct current but also as metallic vacuum lamps and may be used as ordinary Geissler tubes in connection with an induction coil. Finally, the cost of a battery of small cells which brings an existing lighting circuit from 440 volts up to 840 volts, or of a correspondingly small generator, hardly exceeds the cost of a large induction coil.

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[1] EDITORIAL NOTE.—It was the intention of the authors that this bibliography should accompany their first article of this series, published in the *Journal* for June (**31**, 404-418, 1910). Through an oversight in the editorial office, however, that intention was not carried out, and the bibliography is accordingly appended here. See page 405 of the June number.

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SERIES IN THE SPECTRA OF CALCIUM, STRONTIUM, AND BARIUM

By F. A. SAUNDERS

Two abstracts have already appeared[1] of preliminary notes, presented to the Physical Society, on spectrum series in these elements. The following pages give a full account of these series, as far as they are known at present, together with tables of the arc spectra of the elements, which are more nearly complete than any that are already available.

The first series to be found in these spectra were the two main triplet series, first and second subordinate, of which Kayser and Runge gave both for *Ca* and the first for *Sr*. Rydberg[2] found the second *Sr* series, and the writer[3] recently sorted out the same two in *Ba*. One other triplet series in each spectrun is now known, called the "narrow triplets," the lines being closer together than the main triplets. These were found for *Sr* by Fowler,[4] and for *Ca* and *Ba* by the writer.[5] The wave-lengths for the *Ca* series have been measured with accuracy by Barnes,[6] the lines having been obtained sharp by reducing the pressure around the source of light. Without this precaution they are too diffuse to admit of accurate measurement. It was hoped that they could be obtained equally well by the same method in the case of *Sr* and *Ba* also, but the experiment has not yet succeeded, as far as the narrow triplet series are concerned. The lines are relatively fainter than in *Ca*, and fade away with reduction of pressure as fuzzy lines; if present at all in the low-pressure (5 mm) spectra, they have been too faint to impress themselves upon my photographs.

The other series in these spectra may be classified as pair and single-line series. These are considered in detail below.

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[5] *Ibid.*, **21**, 195, 1905, and **23**, 228, 1908.

[6] *Ibid.*, **30**, 14, 1910.

Some of the apparatus employed has been described before.[1] A four-inch grating, purchased through the generosity of the Rumford Fund Committee, was used for the general survey of these spectra. For more exact wave-length measurements beyond λ 2400, I took a set of one-hour arc spectrum photographs with the four-inch concave grating (Rowland mounting, first order) at the Dominion Observatory, Ottawa, Canada. My best thanks are due to Dr. W. F. King, Director of the observatory, for permission to use this instrument, and to Mr. J. S. Plaskett, Chief Astronomer, for many favors in that connection. These spectra have excellent definition and considerable intensity, reaching in one case down to λ 1931.

For still shorter wave-lengths a quartz spectrograph was used, with Schumann plates at first, but it was later found that lantern slide plates were slightly more sensitive to the region in question (λ 2000), and they were accordingly substituted. The ordinary fast photographic plate is distinctly less sensitive than the lantern slide to these rays. The new lines that were found were faint and diffuse, and the wave-length measurements beyond λ 2150 may be considerably in error, possibly as much as a whole unit in the worst cases.

A few spark photographs were taken for comparison, but most of these series are obtained in far greater intensity from the arc, even the pair series, which are *relatively* (not absolutely) strong in the spark; so that the spark photographs were not much used.

A set of photographs was also obtained with the arcs inclosed in an iron box, from which the air could be largely removed by means of a Geryk pump. I am indebted to Mr. O. S. Blakeslee for working out the design of this very convenient apparatus, and constructing it. It is fitted with tubes. Two of these hold the terminals opposite one another, so that they can be turned about, or adjusted along their length, even when the vessel is exhausted. A third tube at right angles to these is sealed by a quartz plate at the farther end, and carries a quartz lens, which throws an image of the arc, outside of the apparatus, on the slit of the spectroscope. The terminals were usually graphite rods, saturated with chlorides

[1] *Astrophysical Journal*, **20**, 188, 1904.

of the elements, but, for *Ca*, I used the metal itself, and for *Ba*, also, on one or two occasions.

SINGLE-LINE SERIES

In the spark spectrum of *Ca* beyond λ 2400 very little is to be found, except some members of the pair series; but, on changing to the arc, a remarkable difference is noted. The pairs are then relatively faint, and an isolated and very obvious series of single lines comes into prominence. These are reversed and unsymmetrically broadened. The first three lines are included in the list of Kayser and Runge. Since my report on this series before the Physical Society, Barnes[1] has also measured the first four of its lines with the arc under reduced pressure. The lines under these conditions, though sharp, become so faint that I have been unable to obtain measurable images of more than five of them. The wavelengths of the outer lines, obtained at normal pressure only, are therefore not very exactly known.

The spectrum of *Sr* shows a precisely similar series, the only important difference noted being that the pair and single-line series are more nearly of the same intensity in this case than in that of *Ca*. Reduction of pressure has the same effect on the *Sr* lines as on those of *Ca*. Table I below shows these new series, under the designation *SL1*.

The spectrum of *Ba* probably contains a similar series, but I have not been able to sort it out. The line λ 2597.26 has exactly the right aspect, and others look promising, but the series seems to be widened out, probably shifted toward the red, and reduced in intensity compared with that in *Sr*.

Barnes has shown that the light of the *Ca* λ 4226.9 comes more strongly from the positive terminal of the arc, while the opposite is the case with the pair-series lines. I have found that *Ca* λ 2398.6 shows the same peculiarity as λ 4226.9. Since the latter is also strongly reversed in the arc, and single, it would be natural to suspect that it was a member of the same series with these new reversed single lines. In the effort to test this, I have taken long-exposure photographs with the quartz spectrograph of an oxy-

[1] *Ibid.*, **30**, 18, 1910.

hydrogen flame colored with calcium, as well as an ordinary Bunsen flame fitted with an attachment for spraying a solution of calcium chloride into it. In neither case did I obtain an image of any ultra-violet lines, though λ 4226 (which is a flame line) was strong. This may possibly be due to too short an exposure, but I am disposed to think that 4226 does not belong to this series, and that the ultra-violet lines are not flame lines. Experiments with the high-temperature oven spectra ought to settle this point easily, since 4226 then has an enormous intensity. A formula which would include 4226 with the others would have to be of an unusual type, if indeed it could be found at all. If, then, 4226 does not belong to this series, it is probably the first member of a principal series of single lines, to which SL_1 is a subordinate, and of which the other lines lie in the Schumann region, and have not yet been found. Similar remarks would apply also to the strong lines Sr λ 4607 and Ba λ 5535.

Other single-line series have also been sorted out. Certain lines in the blue and violet have the aspect of outer series members, and have been long under suspicion, but it was difficult to prove their relationship to one another. This was finally done by making use of the "combination principle" of Ritz.[1] According to this idea, if one series is found, and its formula calculated, other series may reasonably be expected to be found, whose formulae are similar to the first, the differences amounting to simple alterations in one or two constants of the formula. The very simplest alteration possible is to change the "end-constant" only; that is, to shift the series as a whole to some other part of the spectrum without altering the separations of its lines on the wave-number scale. Having already plotted out these spectra on a wave-number scale on long rolls of paper, I had no trouble in searching for such repetitions of the series SL_1, and one was soon found. The data for this series (called SL_2) are given in Table I, together with the differences between the wave-numbers of corresponding lines in the two series. The corresponding series in Sr, which was readily picked out, is also given in the table. It might seem that these two series SL_1 and SL_2 together constitute one

[1] *Physikalische Zeitschrift*, **9**, 521, 1908.

series of very wide pairs. This is unlikely since the appearance of the lines is so different. One set is strong and reversed; the other, weak and not reversed.

When the violet regions of these spectra were again examined, a few lines were noted that looked like series lines but were not included in the previous series. Most of these were seen to belong together to another new series, *SL3*, as shown in Table I. These lines are more diffuse than those of *SL2*, the long wave-length members stronger, the others weaker than those of *SL2*. The members of the two series alternate, just as two subordinate series do. It therefore seems probable that these form a pair of series of the subordinate type, running to the same end. If so, the last series, *SL3*, should be called a "first subordinate" series, from its relative strength and diffuse appearance. *SL3* may have a red member, which I cannot distinguish with certainty. There are several "applicants" for the vacancy in both *Ca* and *Sr*. λ 6718 looks very promising for *Ca*, but the corresponding *Sr* line is hard to select. This point cannot be settled until some observations of the Zeeman effect, or other crucial tests, are obtained for the lines concerned.

The list of lines in *SL3* is very short on account of the faintness of the outer members of this series. The fourth line of *Ca SL3* is due at 3973.3, but cannot be observed on account of the close proximity of the far stronger line 3973.89 which belongs to the second subordinate triplet series.

The wave-lengths for a few lines, given by Kayser and Runge, have been slightly altered. The reason for this is that these lines appear to shift when the pressure is reduced, the shift being measured, of course, with reference to other spectrum lines near by, which are sharp under all circumstances. If, therefore, the true wave-length is desired, the only accurate way of obtaining it is to put the arc *in vacuo*. This has been done, and some new values obtained. In no case was the line *in vacuo* outside of the space occupied by the widened line in air; but the density maximum was shifted in some cases by as much as 0.5 unit. For instance, one of the *Sr* lines has an apparent wave-length at normal pressure, measured by means of some of its strong and sharp neighbors, of

4406.8. When the pressure is reduced to 20 cm, and the wave-length (at normal pressure) deduced from this spectrum, it comes out 4406.45; similarly, from a 5 mm photograph, 4406.29. It appears to be often the case that the wave-lengths of diffuse or broadened lines obtained from arc in air are not accurate, even when measured by the projecting points obtained with an astigmatic grating. This is important in connection with series, as most of the outer series members are of this class and should be located with great accuracy, if series laws are to be studied.

In the table, the observer's name is indicated by initials: K & R signifying Kayser and Runge; EH, Exner and Haschek; B, Barnes, and S, the writer. In case two observers are given together, average values have been used for the wave-lengths. The column W.-N.D. gives the wave-number difference between the line of *SL1* and the corresponding line of *SL2;* that is, the amount by which the two series are separated. The constancy of these numbers is very satisfactory. The value of n in the sixth column is the one used in the formula. The last two columns give the calculated wave-number and the difference between the calculated and observed wave-lengths. The formulae used are presented below:

Ca SL1 and *SL2*

$$1/\lambda = \left.\begin{matrix} 49290.0 \\ 27441.2 \end{matrix}\right\} - \frac{109675}{\left\{ n+0.16+\dfrac{112.444}{n^4}-\dfrac{4937.15}{n^8} \right\}^2}$$

Ca SL3

$$1/\lambda = 27441.2 - \frac{109675}{\left\{ n+0.9344-\dfrac{9.3989}{n^5} \right\}^2}$$

Sr SL1 and *SL2*

$$1/\lambda = \left.\begin{matrix} 45935.0 \\ 25786.0 \end{matrix}\right\} - \frac{109675}{\left\{ n+0.0554+\dfrac{18.6419}{n^2}-\dfrac{124.57}{n^4}+\dfrac{263.26}{n^6} \right\}^2}$$

Sr SL3

$$1/\lambda = 25786 - \frac{109675}{\left\{ n+0.9340+\dfrac{50.149}{n^5} \right\}^2}$$

The series are similar to the one in *Mg* which Rydberg[1] discovered, for which he found a four-constant formula necessary.

TABLE I

Series	Wave-Length	Observer	Wave-No.	W.-N.D.	n	Calc. Wave-Number	C.-O. (λ)
Ca SL1	2398.66	K & R	41677.4	21849.1	3	41677.6	−0.01
	2275.60	K & R	43931.1	21848.4	4	43931.0	+0.01
	2200.80	B. K & R	45424.2	21849.0	5	45425.5	−0.06
	2151.00	B. S	46475.7	21846.3	6	46476.8	−0.05
	2118.99	S	47178.7	21846	7	47177.9	+0.04
	2097.8	S	47654	....	8	47653.8	+0.01
	2083.2	S	47988	....	9	47987.2	+0.03
	2072.8	S	48229	....	10	48229.9	−0.04
Ca SL2	5041.93	K & R	19828.3	....	3	19828.8	−0.12
	4527.17	K & R	22082.7	....	4	22082.2	+0.10
	4240.58	K & R	23575.2	....	5	23576.7	−0.27
	4059.06	S	24629.4	....	6	24628.0	+0.23
	3946.3	S	25333	....	7	25329.1	+0.61
Ca SL3	4878.34	K & R	20493.2	....	3	20493.2	0.00
	4355.41	K & R	22953.6	....	4	22953.6	0.00
	4108.70	S	24331.9	....	5	24330.1	+0.30
	....	....	....	....	6	25161.2	......
	3889.8	S	25701	....	7	25699.3	+0.26
Sr SL1	2931.98	K & R	34096.9	20149.3	2	34096.3	+0.05
	2569.60	EH. S	38905.2	20148.6	3	38905.3	+0.01
	2428.16	EH. S	41171.2	20149.1	4	41170.6	+0.04
	2354.40	EH. S	42460.8	20149.1	5	42460.8	0.00
	2307.5	S	43323	20146	6	43325	−0.06
	2275.5	S	43934	20148	7	43925	−0.4
	2253.5	S	44362	....	8	44349	−0.6
	2237.4	S	44682	....	9	44658	−1.1
	2226.0	S	44910	....	10	44887	−1.1
Sr SL2	7167.7	S	13947.6	....	2	13947.3	+0.15
	5330.00	S	18756.6	....	3	18756.3	+0.08
	4755.59	K & R	21022.1	....	4	21021.6	+0.11
	4480.73	K & R. S	22311.7	....	5	22311.8	−0.02
	4313.38	S	23177.3	....	6	23176	+0.24
	4202.95	S	23786.3	....	7	23776	+1.7
Sr SL3	5156.37	K & R	19388.2	....	3	19388.2	0.00
	4678.39	K & R	21369.0	....	4	21369.0	0.00
	4406.29	S	22688.6	....	5	22688.1	+0.1
	4253.7	S	23503	....	6	23509	−1.1

It is, therefore, not surprising to find that none of the usual three-constant formulae will fit any of them. For the *SL3* series a three-constant formula is used, involving n^5, but only a few terms

[1] *Wiedemann's Annalen,* **50**, 625, 1893.

are known, and a four-constant formula might be required for the entire series.

For the series *SL1* and *SL2* one formula suffices with change only of the end-constant. A four-constant formula was found necessary for *Ca*, and for *Sr* I have not been able to obtain an expression with less than five adjustable constants to satisfy the nine observed values within experimental error. If no attention were paid to the first two or three lines, it could be done with three constants, and this may be looked upon as an argument against including the lines in question; but their appearance, relative intensities, and changes under altered conditions (arc to spark, reduced pressure, etc.), as well as the constancy of the numbers under the W.-N.D. column, all point strongly to including them in the series.

The difficulty with the formula in such a case is clearly seen by examining the various values that the quantity $\sqrt{\dfrac{109675}{a-1/\lambda}}$ takes for each line, *a* being the end-constant. This quantity equals the denominator (not squared) in the formula. If *a* is properly selected, the difference between each of these numbers and the corresponding integer *n* (to which they are generally nearly equal) ought to settle down to approximate constancy for the higher members of the series. It does so for most of the series found by Kayser and Runge. But in the case of *Sr*, *SL1* and *SL2*, these differences are at first rather large, then increase slightly to a maximum, and then drop, fast at first, more slowly later, settling down to their ultimate constant value. If we plot their curve, it has a point of inflection, which the corresponding curves for the simpler series have not. To represent these numbers properly, we must use an algebraic expression involving at least three constants, or more. It does not improve matters in this case to alter the "universal constant" 109,675; while the separate series can be represented equally well in this way (though no better), it becomes impossible to make the two series *SL1* and *SL2* end together. As it seems likely a priori that they have the same end-constant, I have abandoned this line of attack. Nor are we much better off if we examine the differences between the denominator squared

and n^2. The same general form in the curve above mentioned is prominent in the case of the *Mg* series of single lines that Rydberg discovered, as well as elsewhere; so that one is tempted to look upon it as a general property of these series. It may still be possible to find a four-constant formula for all of them, and of the same type for all, but the search is laborious, and it is not unlikely that the result will be first reached (if at all) from theoretical investigations, rather than from the experimental side.

It is apparent that one of the constants (0.9344) for *Ca SL3* is practically the same (could easily be adjusted to be the same exactly) as the corresponding constant for *Sr SL3*. This agreement may be accidental, but it appears to be another of the many likenesses between the spectra of these two elements.

A further application of the combination principle is to be noted. The end-constants of *SL1* and *SL2* appear to be intimately related, as the following will show. According to Rydberg's simple theory, the end-constant itself takes the form $\dfrac{N}{(n+b)^2}$. If we consider this correct, as a first approximation, and put this fraction equal to 49,290, the end-constant for *Ca SL1*, we find that the end-constant for *SL2* is given by the expression $\dfrac{N}{(n+b+0.5)^2}$, the error being only 200 out of 27,441. A similar result is obtained for *Sr*. That the agreement is not exact is due to the fact that the expression $n+b$ is too simple, and must be enlarged by the addition of other terms, involving n and other constants. If n is increased by the addition of 0.5 again, we might hope in like manner to find the end-constant of another new series. This would then end near $\lambda\,5700$ ($1/\lambda = 17,500$) for *Ca*, and if such a series exists it might begin with the great line found by Paschen at $\lambda\,10,345$, but I have not succeeded in sorting it out. In *Sr* the necessary ultra-red observations are lacking for such a search.

THE PAIR SERIES

It has been known for some time, from observations of the Zeeman effect,[1] that there were series of pairs in the spectra of *Ca*, *Sr*, and *Ba*. Only one pair was first found for each of these,

[1] *Astrophysical Journal*, 16, 123, 1902.

but Eder and Valenta[1] measured a couple of pairs in *Ca* which (though the authors do not mention it) belong to the same grouping. Ritz[2] recognized this fact and summed up the series as far as known. Since the spectra of *Sr* and *Ba* are very like that of *Ca*, and consist, on the whole, of slower vibrations, it looked like a promising experiment to search for pairs in these series in the extreme ultra-violet, in the hope of finding enough of them to serve as a basis for the calculation of the formulae. This attempt has been partially successful, and the results are given in Table II, below. In the column of observers, H refers to Hermann,[3] and EV to Eder and Valenta.

In *Ca* there are not enough terms for the calculation of the formulae, excepting in the case of the first subordinate series. Here, however, we have only three terms, including the one in the ultra-red, proposed by Paschen,[4] and even a three-constant formula would require all the lines for the determination of its constants. Ritz[5] calculated the formula for this series (without the ultra-red term) from two terms, along with the principal series pair, which his theory connects with this series, and obtained a value for the end-constant. This value is too low to suit the above three terms, since they converge too slowly in the ultra-violet. Other terms of this series were unsuccessfully sought for on my photographs.

For *Sr* we can calculate two formulae as follows, using in each case the line of greatest wave-length in the pair.

First subordinate series

$$Sr\ P_1 \quad 1/\lambda = 59282 - \frac{109675}{\left\{ n - 1.79 + \frac{6.173}{n^2} \right\}^2} \qquad n = 3, 4, 5, \text{etc.}$$

Second subordinate series

$$Sr\ P_2 \quad 1/\lambda = 58782 - \frac{109675}{\left\{ n - 1.4533 + \frac{6.385}{n^2} \right\}^2} \qquad n = 2.5, 3.5, \text{etc.}$$

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[1] *Sitzungsberichte der Wiener Akademie*, 1893; *Beiträge zur Photochemie und Spectralanalyse*, p. 57, 1904.

[2] *Physikalische Zeitschrift*, **9**, 521, 1908.

[3] *Annalen der Physik*, **16**, 684, 1905.   [4] *Ibid.*, **29**, 659, 1909.

[5] *Physikalische Zeitschrift*, **9**, 529, 1908.

TABLE II

Series	Observer	Wave-Length	Wave-Number Difference
Ca, principal..........	K & R K & R	3968.63 3933.83	222.9
Ca, first subordinate ...	H H H	8662.42 8542.48 8498.32	222.8
	K & R K & R K & R	3181.40 3179.45 3158.98	223.1
	. . . . . . EV EV	. . . . 2113.01 2103.47	214.5
Ca, second subordinate .	K & R K & R	3737.08 3706.18	223.0
	EV EV	2208.95 2198.03	224.8
Sr, principal..........	K & R K & R	4215.66 4077.88	801.3
Sr, first subordinate ...	K & R K & R K & R	3475.01 3464.58 3380.89	800.9
	S EH EH	2324.60 2322.47 2282.14	800.1
	. . . . . . S S	. . . . 1995.7 1965.2	778
Sr, second subordinate .	K & R K & R	4305.60 4161.95	801.6
	EH EH	2471.71 2423.67	801.6
	S S	2053.3 2020.5	791
Ba, principal..........	K & R K & R	4934.24 4554.21	1690.7
Ba, first subordinate ...	K & R K & R K & R	4166.24 4130.88 3891.97	1691.0
	K & R K & R EH	2641.51 2634.91 2528.52	1691.0
	S S S	2235.50 2232.90 2154.02	1691.2

TABLE II—*Continued*

Series	Observer	Wave-Length	Wave-Number Difference
	. . . .	. . . .	
Ba, first subordinate . .	S	2355.0	1650
	S	1987.7	
Ba, second subordinate .	K & R	4900.13	
	K & R	4525.19	1690.5
	K & R	2771.51	
	K & R	2647.41	1690.9
	S	2286.21	
	S	2201.07	1691.3
	S	2082.8	
		. . . .	

Some modification should apparently be made in the denominator of these formulae in order to bring the end-constants into agreement, but there are not enough terms known to test the formula. The end-constant derived by Ritz is, as in Ca, much too low.

For Ba four terms are known in each of two series, and, for these, three-constant formulae have been obtained, which give good agreement for all the lines. The line of greater wave-length in each group has been used for calculation, as before.

First subordinate series

$$Ba \ P_1 \quad 1/\lambda = 58220 - \frac{390431}{\{n+0.37758\}^2} \quad n = 3, 4, 5, \text{etc.}$$

Second subordinate series

$$Ba \ P_2 \quad 1/\lambda = 58425 - \frac{410890}{\{n+0.2873\}^2} \quad n = 3, 4, \text{etc.}$$

These may be considered to give a fair value for the end of the series, but it is over 8000 higher than the one found by Ritz. It seems, then, that in all three elements the pair series converge less rapidly than the theory of Ritz demands. Three-constant formulae involving the universal constant 109,675 could not be found to suit these Ba series. Four constants were then required.

Runge and Paschen[1] note also a pair of lines in Ba, λ 6497 and λ 5853 (wave-number difference, 1690.6), which have the Zeeman

[1] *Astrophysical Journal*, 16, 123, 1902.

separations that are characteristic of first subordinate lines, but interchanged. They suggest the possibility of these being the first members of a principal series of a new sort, related to the first subordinate series instead of, as usual, to the second. Such a series should have a term or two within reach, but I have not been able to identify any of them.

In all three elements there occurs a strong pair in the ultra-violet: *Ba* λ 2335 and 2304; *Sr* λ 2165 and 2152; *Ca* λ 1840 and 1837, which are reversed in *Sr* and *Ba*, and probably in *Ca* also, and the line of greater wave-length is the stronger in each. They therefore look like subordinate-series pairs in a series of great strength, the rest of which is in the Schumann region. This possibility would, perhaps, not be worth noting, were it not for the fact that the *Ca* spectrum shows a curious "pair" of lines (appearing only with arc at low pressures) near λ 5000, which may be related to these pairs. The one in the green is apparently accompanied by a satellite (very faint), just as first subordinate pairs always are, and the wave-number difference for the group is equal to that for the ultra-violet pair. Similar "vacuum pairs" appear in *Mg* (two in this case), but I have sought for them in *Sr* in vain; they must be very faint, if present at all. The *ratios* of the wave-number differences for these ultra-violet pairs in *Ca*, *Sr*, and *Ba* are nearly the same as for the regular pair series, which are related to the atomic weights. This would indicate that they were not accidental pairs. These facts taken together make it look probable that there are series of narrow pairs, as well as the wider ones, most of the lines being at present inaccessible. These spectra also show a few pairs of the same separation as the first two lines of the main triplets, but there is no obvious relation among them.

In regard to the unclassified lines in these spectra, their number is still fairly large, especially in the case of *Ba*. It is likely that several more series are to be found among them. Paschen[1] gives an ultra-red member of the *Ca* "Principal Series of Triplets" (λ 19,900), and seeks to combine this with lines at λ 9546.8 and 7288.0 into a series. Unfortunately the line λ 7288 is absent from my photographs, and this leaves the series rather short, having at

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[1] *Annalen der Physik*, **29**, 656, 1909.

the most but two terms of its own, and this is the more unfortunate as it should be (by analogy) one of the strongest series in *Ca*. He also suggests several single combinations, after Ritz (*op. cit.*, p. 658), which might lead to new series. Two of them involve lines (λ 6833 and λ 3653) which certainly do not occur on my photographs.

There are, finally, certain curious groups of lines, six to a group, which occur in all three spectra in the same sort of way. These six lines include a triplet of the same separation as the main triplets, a pair of the same separation as the first two lines of the triplet, and a single line. They are usually strong, and reversed. The groups occur in *Ca* near λ 3000 and λ 4300. The corresponding ones in *Sr* are near λ 3350 and λ 4800, and in *Ba* around λ 4500 and in the deep red. These look like some new sort of "complex" triplets, but there is no very obvious relation among them, or with other lines. They are "arc" lines, slightly stronger about the negative pole of a direct-current arc than the positive one, like the triplet series, and unlike the *SL* series; they seem to be physically related to the main triplet series in every way. They furnish the most striking feature among the unclassified lines, and it would be very interesting to find the key to their arrangement. Observations in the Schumann region ought to throw light on this group.

For the sake of completeness, I give, below, tables of the arc spectra of these elements. It is hoped that these are fairly accurate, and contain all the lines known to date. The wave-lengths given are those furnished by the observers mentioned, who are indicated by the same abbreviations as in the tables above, with the addition of the following: *R* for Rowland's table of the sun's spectrum; *P* for Paschen;[1] *J* for Janicki;[2] *K* for Kent;[3] *F* for Fowler,[4] and Ryd. for Rydberg.[5] Rowland's wave-length system is used. The column under *E* contains the error, estimated as well as the available data (sometimes lacking) permit. The inten-

[1] *Annalen der Physik*, **26**, 655, 1909.

[2] *Ibid.*, **29**, 833, 1909.

[3] *Astrophysical Journal*, **28**, 225, 1908.

[4] *Ibid.*, **21**, 81, 1905.

[5] *Wiedemann's Annalen*, **52**, 119, 1894.

CALCIUM ARC SPECTRUM

Intensity	Wave-Length	Observer	E	Series
2	22655.9	P	1.5	
2	24.6	P	2.5	
1	10.0	P	3	
1	19946.8	P	3	TH (?)
2	35.8	P	1	TH (?)
1	17.5	P	3	T1
4	19859.9	P	2	TH (?)
1	17.3	P	2	
3	19777.4	P	1	T1
2	19507.1	P	1	T1
3	19452.9	P	1	T1
2	19310.6	P	1	T1
2	16432.6	P	3	
2	16200.0	P	3	
2	16162.2	P	3	
2	44.8	P	3	
2	13038.0	P	3	
3	12821.6	P	1.5	
10	10345.0	P	1.5	
3	9694.5	P	3	
1	9546.8	P	3	TH (?)
2	9250.8	P	2	
7	8662.42	H	. . .	P1
8	8542.48	H	. . .	P1
6	8498.32	H	. . .	P1
6	7326.45	S	0.3	
6	7202.45	S	0.3	p
8	7148.5	S	0.3	p
1	6798.9	S	0.3	
1	89.6	S	0.3	
2	84.35	S	0.3	
8	18.1	S	0.3	
3	6573.0	S	0.3	
1	09.1	S	0.3	
5	6499.85	K & R	0.1	p
6	93.97	K & R	0.1	p
6	71.85	K & R	0.1	
7 r	62.75	K & R	0.1	
3	55.75	S	0.3	p
6	49.99	K & R	0.1	p
8 r	39.36	K & R	0.1	
1	17.95	S	0.3	
1	6395.4	S	0.3	
2	6261.7	S	0.3	
6	6169.87	K & R	0.1	
4	69.36	K & R	0.1	
4	66.75	K & R	0.1	
4	63.98	K & R	0.1	
8 r	62.46	K & R	0.1	T2
2	61.60	K & R	0.1	
7 r	22.46	K & R	0.05	T2
6 r	02.99	K & R	0.05	T2
2 hR	5867.94	K & R	0.1	
8 h	57.77	K & R	0.1	

CALCIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
6	5603.06	K & R	0.05	
6	01.51	K & R	0.05	
6 h	5598.68	K & R	0.05	
8	94.64	K & R	0.05	
6	90.30	K & R	0.05	
8	88.96	K & R	0.05	
9	82.16	K & R	0.05	
6	13.07	K & R	0.1	
8	5349.66	K & R	0.05	
1	42.4	S	0.2	
8	5270.45	K & R	0.05	
8	65.79	K & R	0.05	
6	64.46	K & R	0.05	
6	62.48	K & R	0.05	
6	61.93	K & R	0.05	
4	60.58	K & R	0.05	
7	5189.05	K & R	0.05	
7 hR	5041.93	K & R	0.05	SL2
0	21.45	S	0.1	vp
2	20.35	S	0.1	vp
1	01.87	S	0.1	vp
8 hR	4878.34	K & R	0.1	SL3
3 hV	47.22	K & R	0.2	
2 hV	33.85	K & R	1.0	
2 hV	23.04	K & R	1.0	
2 hV	07.47	K & R	1.0	
4 hV	4685.40	K & R	0.5	
1 hV	24.71	K & R	0.5	
2	4586.22	J	0.1	t
7	86.12	K & R	0.1	t
6	81.66	K & R	0.1	t
5	78.82	K & R	0.1	t
4	27.17	K & R	0.1	SL2
1 h	12.73	K & R	1.0	
1 h	09.89	K & R	1.0	
1 h	08.04	K & R	1.0	
4	4456.81	K & R	0.03	T1
6 r	56.08	K & R	0.03	T1
8 r	54.97	K & R	0.03	T1
7 r	35.86	K & R	0.03	T1
7 r	35.13	K & R	0.03	T1
7 r	25.61	K & R	0.03	T1
3 hR	4355.41	K & R	0.1	SL3
6 r. hV	18.80	K & R	0.03	T
6 r	07.91	K & R	0.03	
7 r	02.68	K & R	0.03	p
5 hV	4299.14	K & R	0.03	T
6 r. hV	89.51	K & R	0.03	T
6 r	83.16	K & R	0.03	p
2 h	40.58	K & R	0.1	SL2
10 r	26.91	K & R	0.03	?
1 h	4108.70	S	0.1	SL3
3 hR	4098.66	B	0.1	t
2 hR	95.05	B	0.1	t

Analogy indicates that *Ca* λ 4581 is double; separation about .03 t-m.

CALCIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
2 hR	4092.76	B	0.1	t
1 h	59.06	S	0.1	SL2
7 hR	3973.89	K & R	0.05	T2
10 r	68.63	K & R	0.03	PH
6 hR	57.23	K & R	0.05	T2
1 h	53.6	S	0.2	
5 hR	49.09	K & R	0.05	T2
0 h	46.3	S	0.2	SL2
10 r	33.83	K & R	0.03	PH
1 h	3889.8	S	0.2	SL3
3 h	75.85	B	0.1	t
2 h	72.60	B	0.1	t
2 h	70.57	B	0.1	t
2 h	3753.56	B	0.1	t
1 h	50.40	B	0.1	t
1 h	48.39	B	0.1	t
7	37.08	K & R	0.03	P2
6	06.18	K & R	0.03	P2
1 h	3678.46	B	0.1	t
1 h	75.53	B	0.1	t
1 h	73.49	B	0.1	t
4 r. hR	44.86	K & R	0.05	T1
8 hR	44.50	K & R	0.05	T1
7 hR	31.10	K & R	0.05	T1
7 r. hR	30.83	K & R	0.05	T1
7 r. hR	24.15	K & R	0.05	T1
5 hR	3487.76	K & R	0.05	T2
4 hR	74.98	K & R	0.05	T2
4 hR	68.68	K & R	0.05	T2
7 hV	3361.92	K & R	0.1	T1
6 hV	50.22	K & R	0.1	T1
5 hV	44.49	K & R	0.1	T1
4 hR	3286.26	K & R	0.1	T2
3 hR	74.88	K & R	0.1	T2
3 hR	69.31	K & R	0.1	T2
4 hV	25.74	K & R	0.5	T1
3 hV	15.15	K & R	0.5	T1
3 hV	00.68	K & R	0.5	T1
4	3181.40	K & R	0.03	P1
7	79.45	K & R	0.03	P1
1 h	70.23	K & R	0.5	T2
1 vh	66.95	K & R	2.0	T2
6	58.98	K & R	0.03	P1
2 h	50.85	K & R	0.5	T1
2 h	40.91	K & R	0.5	T1
1 h	36.09	K & R	0.5	T1
1 vh	17.74	K & R	1.0	T2
1 vh	07.96	K & R	1.0	T2
1 vh	01.87	K & R	1.0	T1
5	3009.30	R	0.05	T
6	06.98	R	0.05	p
4	00.91	R	0.05	
4	2999.75	R	0.05	T
5	97.44	R	0.05	p

CALCIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
5	2995.06	R	0.05	T
7	2398.66	K & R	0.1	SL1
2	73.3	S. EV	0.3	
1 h	54.7	S	0.3	
1 h	35.0	S	0.3	
1	2287.9	S	0.3	
5 r. h	75.60	K & R	0.1	SL1
1	49.8	S	0.3	
1	16.7	S	0.3	
2	08.95	EV	....	P2
3 r. h	00.80	B. S	0.2	SL1
2	2198.03	EV	...	P2
2 r. h	51.00	B. S	0.2	SL1
2 h	18.99	S	0.3	SL1
2	13.01	EV	...	P1
2	03.47	EV	...	P1
1 vh	2097.8	S	0.3	SL1
1 vh	83.2	S	0.5	SL1
1 vh	72.8	S	1.0	SL1
4	1839.8	S	1.0	vp
3	37.1	S	1.0	vp

STRONTIUM ARC SPECTRUM

Intensity	Wave-Length	Observer	E	Series
6	7309.7	S	0.3	
5	7232.6	S	0.3	
7	7167.7	S	0.2	SL2
8 .	7070.7	K & R	0.2	T2
6	6892.8	S	0.2	
8	78.8	K & R	0.2	T2
7	6791.4	K & R	0.2	T2
5	6643.8	S	0.2	
6	17.6	S	0.2	
8	6550.53	K & R	0.2	p
6	47.1	S	0.2	
1	21.3	S	0.2	
8	04.17	K & R	0.1	
2	6466.1	S	0.2	
2	47.0	S	0.2	
8	08.65	K & R	0.1	
7	6388.4	S	0.2	
8	86.74	K & R	0.1	p
7	80.95	K & R	0.1	
5	70.2	S	0.2	
4	64.3	S	0.2	
2	45.9	S	0.2	
4 hR	5970.38	K & R	0.05	
1	5848.01	K & R	0.1	
2	17.01	K & R	0.05	
3	5767.29	K & R	0.1	
3	5693.35	S	0.1	
3	74.25	S	0.1	

STRONTIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
3	5556.66	S	0.1	.
6 h	43.49	K & R	0.05	
7	40.28	K & R	0.05	
9	35.01	K & R	0.05	
7	22.02	K & R	0.05	
8	04.48	K & R	0.05	
7	5486.37	K & R	0.05	
8	81.15	K & R	0.03	
7	51.08	K & R	0.03	
6	5330.00	S	0.1	SL2
7	5257.12	K & R	0.03	
7	38.76	K & R	0.03	
6	29.52	K & R	0.03	
6	25.35	K & R	0.03	
6	22.43	K & R	0.03	
4	13.23	K & R	0.03	
2 hR	5166.2	S	0.1	
8	56.37	K & R	0.05	SL3
3	4971.85	K & R	0.05	T1
6	68.11	K & R	0.03	T1
9 r	62.45	K & R	0.03	T1
7	4892.87	Ryd	0.03	t
7	92.20	K & R	0.03	t
7	76.49	Ryd	0.05	T
7 r	76.23	Ryd	0.05	T1
8 r	72.66	K & R	0.05	T1
6 h	69.41	K & R	0.05	t
6 h	68.92	K & R	0.03	t
5 h	55.27	K & R	0.05	t
7 r	32.23	K & R	0.03	T1
7 r	12.01	K & R	0.03	p
6	4784.43	K & R	0.05	T
4	55.59	K & R	0.1	SL2
6	42.07	K & R	0.03	T
3 h	29.93	K & R	0.1	
6	22.43	K & R	0.03	p
2	14.2	S	0.1	
1	07.4	S	0.1	
1	04.2	S	0.1	
1	4688.8	S	0.1	
1	84.6	S	0.1	
5 h	78.39	K & R	0.1	SL3
10 r	07.52	K & R	0.03	SL?
3	4531.54	K & R	0.03	
3 h	4480.73	K & R. S	0.1	SL2
5 h	38.22	K & R	0.03	T2
3	12.82	K & R	0.03	
3 hR	06.29	S	0.1	SL3
5 h	4361.87	K & R	0.03	T2
6 hR	38.00	K & R	0.05	t
4	26.60	K & R	0.03	T2
5 hR	19.39	K & R	0.05	t
2 h	13.38	S	0.1	SL2
4 h R	08.49	K & R	0.1	t

STRONTIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
6	4305.60	K & R	0.1	P₂
1 hR	4253.7	S	0.3	SL₃
10 r	15.66	K & R	0.03	PH
1 h	02.95	S	0.1	SL₂
5	4161.95	K & R	0.03	P₂
2	40.50	S	0.1	
2 h	4087.67	F	....	t
10 r	77.88	K & R	0.03	PH
2 h	71.01	F	....	t
2 h	61.21	F	....	t
1 hR	51.1	S	0.2	
3	33.25	K & R	0.05	T₁
5 hV	32.51	K & R	0.05	T₁
7 hV	30.45	K & R	0.05	T₁
5 h	3970.15	K & R	0.05	T₁
5	69.42	K & R	0.05	T₁
2 h	62.75	S	0.2	
2 h	50.96	F	....	t
4 hV	40.91	K & R	0.05	T₁
1 h	35.33	F	....	t
1 h	26.27	F	....	t
0 h	3867.3	S	0.3	
4	65.59	K & R	0.1	T₂
4	07.51	K & R	0.1	T₂
3	3780.58	K & R	0.1	T₂
7 vh	05.88	K & R	0.2	T₁
5 h	3653.90	K & R	0.1	T₁
6 h	53.32	K & R	0.1	T₁
5 h	29.15	K & R	0.1	T₁
3	28.62	K & R	0.1	T₂
2	3577.45	K & R	0.1	T₂
2	53.6	S	0.1	T₂
3 vh	47.92	K & R	0.3	T₁
2 vh	04.70	K & R	0.3	T₂
3 h	3499.40	K & R	0.2	T₁
2 h	77.33	K & R	0.1	T₁
3 h	75.01	K & R	0.05	P₁
6	64.58	K & R	0.03	P₁
2 vh	57.70	K & R	0.2	T₁
1	56.78	K & R	0.2	T₂
1	34.95	K & R	0.5	T₂
1 h	11.62	K & R	0.5	T₁
1 h	00.39	K & R	1.0	T₁
1 h	3390.00	K & R	0.5	T₁
5	80.89	K & R	0.2	P₁
1	71.15	EH	....	
4	66.43	K & R	0.03	T
5 r	51.35	K & R	0.05	p
3	30.15	K & R	0.03	
3	22.32	K & R	0.03	T
3 r	07.64	K & R	0.05	p
3	01.81	K & R	0.05	T
1 h	3200.4	K & R	0.2	
1 h	3199.1	K & R	0.2	

STRONTIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
1 vh	3190.1	K & R	0.2	
1 vh	89.4	K & R	0.2	
1 vh	82.4	K & R	0.2	
1 vh	72.3	K & R	0.2	
4	2931.98	K & R	0.03	SL1
3 r. h	2569.60	EH. S	0.1	SL1
1	49.62	S	0.1	
2	2471.71	EH. S	0.1	P2
2 h	28.16	EH. S	0.1	SL1
1	23.67	EH. S	0.1	P2
1 r. h	2354.40	S	0.1	SL1
1	24.60	S	0.1	P1
2	22.47	EH. S	0.1	P1
1 h	07.5	S	0.2	SL1
2	2282.14	EH. S	0.2	P1
1 h	75.5	S	0.3	SL1
1 h	53.5	S	0.5	SL1
1 h	37.4	S	1.0	SL1
1 h	26.0	S	1.0	SL1
3 r	2166.11	EH. S	0.2	vp?
2	52.82	EH. S	0.2	vp?
1 h	2053.3	S	0.5	P2
0 h	20.5	S	0.5	P2
0 h	1995.7	S	1.0	P1
0 h	65.2	S	1.0	P1

BARIUM ARC SPECTRUM

Intensity	Wave-Length	Observer	E	Series
2	8915.19	H	0.4	
1	8861.40	H	0.3	
0	8799.86	H	0.3	T?
1	8654.33	H	0.3	
1	8560.20	H	0.3	p
6	8210.73	H	0.3	
0	8148.32	H	0.4	T?
0	20.84	H	0.3	
0	7961.23	H	0.2	p
0	39.21	H	0.2	
4	11.53	H	0.3	T?
7	06.13	H	0.2	T2
0	7878.13	H	0.2	
1	39.57	H	0.2	
5	7780.70	H	0.1	
2	51.92	H	0.1	
1 h	09.96	H	0.1	
1	06.82	H	0.1	
5	7672.42	H	0.1	
3	43.42	H	0.1	
1 h	37.47	H	0.1	
2	10.74	H	0.1	
6	7488.38	H	0.1	
6	60.27	H	0.1	

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BARIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
6	7417.80	H	0.1	
1	10.6	S	0.3	
7	7392.83	H	0.1	T2
4	76.10	H	0.1	
3	59.67	H	0.1	
10	7280.58	H	0.1	
10	29.40	H	0.2	
2	13.83	H	0.1	
2	08.50	H	0.1	
9	7195.71	H	0.2	T2
5	53.72	S	0.3	
8	20.73	H	0.2	
9	7090.51	H	0.1	
1	69.64	H	0.1	
10 r	60.26	H	0.1	
1	6989.82	H	0.3	
3	87.27	H	0.3	
0 h	62.77	H	0.3	
2	6874.35	H	0.1	
4	68.04	H	0.1	
8	65.93	H	0.1	
4	6772.07	H	0.1	
1	61.90	H	0.1	
7	6694.2	S	0.2	
7	75.30	K & R	0.2	
4	54.4	S	0.2	
8	6595.55	K & R	0.1	
3	81.0	S	0.2	
7	27.56	K & R	0.1	
7	6498.93	K & R	0.1	
10 r	97.07	K & R	0.1	
7	83.10	K & R	0.1	
3	51.05	K & R	0.1	
7	6341.88	K & R	0.1	
2 vh	23.0	S	0.5	
2 vh	6258.7	S	0.5	
2 vhR	33.9	S	0.5	
10 r	6141.93	K & R	0.03	
6	11.01	K & R	0.03	
1	6083.63	K & R	0.15	
5	63.33	K & R	0.03	
4	19.69	K & R	0.03	
4	5997.31	K & R	0.03	
1	78.72	K & R	0.20	
4	71.94	K & R	0.03	
3 h	65.06	K & R	0.20	
1 h	62.8	S	0.2	
4	07.88	K & R	0.05	
8 r	5853.91	K & R	0.1	
7 r	26.50	K & R	0.03	
4 h	19.21	K & R	0.05	T1
5 r	05.86	K & R	0.05	
7 r	00.48	K & R	0.05	T1
3 h	5784.24	K & R	0.15	

BARIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
9 r	5777.84	K & R	0.03	T1
1 h	21.66	K & R	0.3	
3 hR	13.62	K & R	0.15	
3 hR	09.82	K & R	0.15	
4 r	5680.34	K & R	0.05	
1 h	64.5	S	0.2	?Ba
1 h	40.8	S	0.2	?Ba
4 vhR	20.41	K & R	0.5	
3 h	5593.45	K & R	0.1	
7	36.07	K	0.05	T1
10 r	35.69	K & R	0.05	SL?
9 r	19.37	K & R	0.05	T1
4 h	5473.94	K & R	0.1	
4 h	37.66	K & R	0.1	
9 r	24.82	K & R	0.05	T1
4 h	05.2	S	0.2	?Ba
1 hR	5393.47	K & R	0.1	
1 hR	81.25	K & R	0.5	
1 hR	79.05	K & R	1.0	
2 hR	65.46	K & R	0.5	
2 hR	09.20	K & R	0.3	
2	05.99	K & R	0.2	
4 hR	02.97	K & R	0.2	
3	5294.40	K & R	0.1	
2 h	91.16	K & R	0.2	
1 h	79.72	K & R	0.2	
3 vh	77.84	K & R	0.15	
5 vh	67.20	K & R	0.1	
1 vh	53.94	K & R	0.1	
2 h	5177.60	K & R	0.05	
3 h	75.74	K & R	0.05	
4 h	60.27	K & R	0.1	
3 h	5055.12	K & R	0.1	
3 h	4947.50	K & R	0.1	
10 r	34.24	K & R	0.03	PH
4 hR	03.11	K & R	0.05	T2
8	00.13	K & R	0.05	P2
4 vh	4877.99	K & R	0.15	
1 vh	07.6	S	0.2	
1 h	4739.40	S	0.2	
1 h	32.53	S	0.2	
6 r	26.63	K & R	0.05	
1 h	24.98	K & R	0.1	
5 hR	00.64	K & R	0.05	T2
2 h	4698.30	S	0.2	
6 r	91.74	K & R	0.03	T
6 hV	73.69	K & R	0.05	
1 h	70.95	S	0.2	
1 h	68.76	S	0.2	
1 h	63.80	S	0.2	
1 h	51.60	S	0.2	
1 hR	42.38	K & R	0.1	
2 hR	36.80	K & R	0.1	
5 hV	28.45	K & R	0.05	

BARIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
1 h	4624.38	S	0.2	
4 hR	20.19	K & R	0.05	T2
3 hV	05.11	K & R	0.05	
5 hR	00.02	K & R	0.05	
3 hV	4591.88	K & R	0.05	
3 hV	89.82	K & R	0.05	
6 r	79.84	K & R	0.03	p
5 r	74.08	K & R	0.05	
10 r	54.21	K & R	0.03	PH
5	25.19	K & R	0.05	P2
5 r	23.48	K & R	0.1	
5	06.11	K & R	0.1	T
5 hV	4493.82	K & R	0.2	T1
6 hV	89.50	K & R	0.5	T1
4	67.36	K & R	0.05	
5 r	32.13	K & R	0.03	T
3	13.96	K & R	0.05	
4 hR	07.10	K & R	0.05	
6 r	02.75	K & R	0.05	p
1 vh	4376.05	S	0.2	
3	59.80	K & R	0.05	
7 r	50.49	K & R	0.1	
5 hV	33.04	K & R	0.05	T1
4	25.38	K & R	0.05	
5 hV	23.15	K & R	0.05	T1
4	4291.32	K & R	0.05	
7 r	83.27	K & R	0.03	
5 h	64.45	K & R	0.05	T1
1 vh	52.6	S	0.2	
3 hR	42.83	K & R	0.05	
4 hR	39.91	K & R	0.05	T2
4	24.11	K & R	0.05	
3 hR	4179.57	K & R	0.2	
5	66.24	K & R	0.05	P1
4	32.60	K & R	0.05	
8 r	30.88	K & R	0.05	P1
2	10.46	K & R	0.1	
2 vh	4087.53	K & R. S	0.1	T1
3 vh	84.94	K & R. S	0.1	T1
2 h	81.1	K & R. S	0.1	
2	26.57	S	0.1	T2
4	3995.92	K & R	0.1	t
8 r	93.60	K & R	0.03	t
2 hR	75.55	K & R	0.1	T2
1 vh	47.6	S	0.2	T1
2 vh	45.6	S	0.2	T1
4	38.09	K & R	0.05	t
7 r	35.87	K & R	0.05	t
3	17.42	K & R	0.05	
7 r	10.04	K & R	0.05	t
2 h	06.20	K & R	0.05	
2 h	00.54	K & R	0.05	
1 h	3895.2	S	0.2	T1
5 h	92.93	K & R	0.1	

BARIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
9 h	3891.97	K & R	0.05	P1
5	89.45	K & R	0.05	
4 h	61.87	K & R	0.15	
1 h	41.72	S	0.2	T2
4 h	3794.77	K & R	0.2	
0 vh	87	S	1.0	T1
0 vh	67.5	S	1.0	T1
2 h	01.87	K & R	0.15	
4 hR	3689.28	K & R	0.15	
2 h	64.76	K & R	0.1	
4	62.62	K & R	0.05	
2	40.72	S	0.1	
3 vh	37.10	K & R	0.5	
7	30.84	S	0.1	
4 hR	11.17	K & R	0.1	
4	3599.60	K & R	0.05	
4 hR	93.58	K & R	0.05	
3 h	88.33	K & R	0.1	
3	86.64	K & R	0.1	
6 r. h	79.97	K & R	0.1	
4 h	77.79	K & R	0.05	
3	76.20	K & R	0.1	
1 h	72.0	S	0.2	
3 h	66.90	K & R	0.05	
3 h	62.23	K & R	0.1	
1 h	52.5	S	0.2	
3 h	48.14	S	0.05	
6 r. h	44.94	S	0.1	
2 vh	31.8	S	0.2	
2	29.6	S	0.2	
5 r. h	25.23	K & R	0.1	
8 r	01.29	K & R	0.05	
1 vh	3464.5	S	0.3	
5 r. hR	20.48	K & R	0.1	t
4 r. hR	3377.19	K & R	0.1	t
3 r. hR	57.00	K & R	0.1	t
3 hR	23.06	K & R	0.1	
1 h	15.90	K & R	0.1	
1 hR	3298.25	K & R	0.1	
3 hR	81.96	K & R	0.1	
3 hR	62.57	K & R	0.1	
2 vh	23.11	K & R	0.2	t
1 vh	04.09	K & R	0.2	
1 vh	3184.45	K & R	0.3	t
0 vh	74	S	1.0	
0 vh	65	S	1.0	
0 vh	3119.48	K & R	0.5	t?
1 h	08.37	K & R	0.5	
7 r	3071.71	K & R	0.03	
4 h	2785.22	K & R	0.15	P2
4 .	71.51	K & R	0.1	
2	39.12	S	0.1	
3 r	02.78	S	0.03	
3	2647.41	S	0.05	P2

BARIUM ARC SPECTRUM—*Continued*

Intensity	Wave-Length	Observer	E	Series
1 h	2646.65	S	0.1	
2 h	41.52	K & R	0.05	P1
6	34.91	K & R	0.05	P1
2 r. vh	2597.26	K & R. S	0.2	
o vh	92.40	S	0.2	
o vh	44	S	2.0	
5	28.52	EH	. . .	P1
1 h	∞.27	S	0.1	
o h	2473.20	S	0.1	
5	2347.67	K & R	0.05	
9 r	35.33	K & R	0.05	vp?
8 r	04.32	K & R	0.05	vp?
2	2286.21	S	0.1	P2
3	54.80	K & R	0.1	
2	45.72	K & R	0.1	
o	35.50	S	0.1	P1
2	32.79	S	0.1	P1
2	14.79	S	0.1	
o	01.07	S	0.2	P2
1	2154.02	S	0.3	P1
o	2082.8	S	1.0	P2
1	55.0	S	1.0	P1
o	1987.8	S	1.0	P1

sities are on a scale of 10 to o, subject to the usual inaccuracies. The abbreviations in the first column are: *h* for hazy; *vh*, very hazy; *hR*, broadened on the long-wave side; *hV*, broadened on the short-wave side; and *r*, reversed. The abbreviations under the column headed "Series" are as follows: *TH*, (possible) principal series (Hauptserie) of main triplets; *T1*, first subordinate series of main triplets; *T2*, second subordinate series of the same; *t*, narrow triplets; *T*, triplets occurring in the groups of six; *PH*, principal series of pairs; *P1*, first subordinate series of pairs; *P2*, second subordinate series of pairs; *p*, pairs occurring in the groups of six; *vp*, the "vacuum pair" in *Ca*, and those which there is reason to believe are related to it; *SL1*, the ultra-violet series of single lines; *SL2*, the related series in the visible spectrum; *SL3*, the single line series, alternating with *SL2*.

Bands are prominent in these spectra, especially at low pressures, but they are omitted from the tables.

SYRACUSE UNIVERSITY
    August 1910

THE BRIGHTNESS OF HALLEY'S COMET AS MEASURED WITH A SELENIUM PHOTOMETER

By JOEL STEBBINS

Anticipating that Halley's comet would become a brilliant object when near the earth, I planned to measure its light with the selenium photometer which is in regular use at this observatory. Visual estimates of the total light of a comet are necessarily rough, and the advantage of the selenium cell lies in the fact that a comet may be compared with the extra-focal image of a star, and the relative light found from galvanometer deflections.

I have in preparation a paper which describes in detail the methods and precautions necessary in observing with selenium. Let it suffice to state here that the apparatus used for the comet includes a selenium cell attached to the 12-inch refractor, the cell being placed 4 inches (10 cm) outside the focus of the large objective. The cell has an effective aperture of $\frac{3}{4}$ inch (18 mm), but the field is limited by a diaphragm, which it is found admits all the light over a circle of about 7 minutes of an arc in diameter. This is nearly one-fourth of the moon's apparent diameter, and it is probable that eye estimates of the comet's brightness never included a larger area. Under the conditions of the observations, a 10-second exposure to a 3.0 magnitude star gives a galvanometer deflection of approximately 3.5 mm, and the probable error of a single deflection is of the order of 10 per cent or 0.35 mm. For brighter stars the deflections are increased, but the errors remain about the same, so that the relative accuracy becomes much greater.

As is well known, the color sensibility of selenium is different from that of the eye, being especially great near the red end of the visible spectrum. In a previous number of this *Journal*,[1] I have already published a determination of the color sensibility of the cell in question, Giltay No. 93; and unless the comet had a most peculiar spectrum, I think it probable that this selenium cell gives the visual magnitudes of the total light of the comet with less systematic error than is present

[1] **27**, 183, 1908.

in the eye comparison of a luminous surface with a point source of light.

In observing, the usual procedure was to take alternate exposures on the comet and comparison star, at intervals of about 1 minute, which is necessary to allow the cell time for recovery. In all of the early morning observations there was a bright sky background, and the exposures were made in the order: sky, star, sky, comet, sky, etc., until the effect from the sky became predominant. When the moon was bright, similar precautions were necessary, and of course in the reductions the extra light of the sky was allowed for.

Although the effect of differential atmospheric absorption may be practically eliminated under proper conditions, it was not possible to do this in the comet observations. The correction for absorption is based upon the Potsdam tables,[1] and I allowed for the increase of absorption on poor nights by multiplying by an arbitrary factor, which was determined from the general appearance of the sky at the time of the observations. This is, of course, only a guess at the true value of the absorption, but a check is secured in the size of the galvanometer deflections given by the same star on different nights.

In the following table are given the results of the observations, the headings of the different columns being self-explanatory. The magnitudes of the comparison stars are from Vol. **50** of the *Harvard Annals*. The difference of magnitude between the comet and star follows from the ratio of the observed galvanometer deflections.

The results of this paper are shown in the column of adopted magnitudes, the values on three nights, when the atmospheric conditions were poor, being in parentheses. Unfortunately, the only really clear nights between May 1 and 24 were those of May 3, 4, 17, and 18, the head of the comet not being visible on the latter two dates. Smoke from forest fires spoiled what would otherwise have been a clear sky. On May 20, a few readings were taken in what seemed to be perfectly clear spaces between passing clouds, but there was no opportunity to use a comparison star. On this date my direct visual estimate of the comet was magnitude 1.0. Not enough material is here available to exhibit the variation of the comet's brightness from night

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[1] Müller, *Photometrie der Gestirne*, p. 515.

OBSERVATIONS OF HALLEY'S COMET WITH THE SELENIUM PHOTOMETER

Date 1910	G.M.T.	Exposures		Absorption Factor	Comparison Star	Magnitude of Star	Diff. of Mag. Comet − Star	Absorption	Magnitude of Comet		Remarks
		Comet	Star						Observed	Adopted	
May 3	21h 37m	9	7	1	γ Pegasi	2.87	−0.74	−0.14	1.99	2.0	Sky perfect
4	21 41	5	4	1	γ Pegasi	2.87	−0.95	−0.07	1.85	1.8	Sky thick and smoky
9	21 39	4	4	2	γ Pegasi	2.87	−1.25	−0.42	1.20	(1.2)	Sky still worse
11	21 31	6	7	3	γ Pegasi	2.87	−1.26	−1.03	0.56	(0.6)	Between clouds, compari-
20	14 4	4	..	3	β Geminorum	1.21	+0.79	−0.02	1.08	(1.1)	son star on May 24
24	14 38	6	4	1	β Geminorum	1.21	+0.64	−0.03	1.82	1.8	
25	15 36	5	4	1.5	ζ Hydrae	3.30	−0.74	+0.07	2.63	2.6	Sky only fair
26	15 30	6	2		ζ Hydrae	3.30	−0.81	+0.10	2.50		
26	15 39	..	5	1	ο Leonis	3.76	−0.71	−0.21	2.84		
26	15 39	..	2	1	α Leonis	1.34	+1.68	−0.34	2.68	2.7	
29	14 48	5	3	1	ζ Hydrae	3.30	+0.27	+0.15	3.72		
29	15 19	9	12	1	ο Leonis	3.76	+0.20	−0.11	3.45	3.5	
30	15 51	11	12	1	ο Leonis	3.76	−0.04	−0.07	3.63	3.6	
31	15 9	13	12	1	ο Leonis	3.76	−0.36	−0.05	3.35	3.4	Cell irregular
June 1	15 35	15	12	1	ο Leonis	3.76	−0.15	−0.05	3.56	3.6	

to night, but it is possible that these measures will be of use for comparison with the work of other observers. As determined with the selenium photometer, Halley's comet became brighter than first magnitude, but at no time when we could observe it did it reach magnitude o.o.

I am indebted to Mr. Percy F. Whisler for his assistance in reading the galvanometer throughout all of the observations.

MINOR CONTRIBUTIONS AND NOTES

VARIABLE RADIAL VELOCITIES OF TWO STARS IN THE *TAURUS* STREAM

The following two stars were included among six reported as having variable velocities by Professor Frost[1] at the Baltimore meeting of the American Association in 1909. The measures have been made on plates taken with the Bruce spectrograph with one prism.

B.D. $15°637$ *Tauri* ($\alpha = 4^h 25^m$; $\delta = +15° 59'$; Mag. $= 5.0$)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
1B 1822.....	1908 Nov. 2	$21^h 57^m$	B	7	$+41$ km	Good
1878.....	Nov. 30	20 34	B	6	32	Good
1882.....	Dec. 4	14 15	F, B	14	30	Good
1911.....	Dec. 21	15 43	L	15	50	Good
1945.....	1909 Jan. 1	16 52	B	13	49	Good
2143.....	Sept. 20	20 26	B	6	34	Weak

B = Barrett; F = Frost; L = Lee.

The spectrum of this star, as well as the following one, is of Vogel's type Ia2, or A5 according to the system adopted in the Harvard Revision.

90 c_1 *Tauri* ($\alpha = 4^h 38^m$; $\delta = +12° 19'$; Mag. $= 4.7$)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
1B 1855.....	1908 Nov. 13	$18^h 10^m$	B	20	$+32$ km	Good
1877.....	Nov. 30	19 21	B	20	62	Good
1883.....	Dec. 4	15 27	B	11	52	Good
1903.....	Dec. 14	19 38	B	24	85	Good

Evidence for the occasional duplicity of many of the lines in these two stars seems almost conclusive, but consistent results for the two components have not as yet been obtained. The reliability of the measures is somewhat impaired by this apparent complexity.

STORRS B. BARRETT

YERKES OBSERVATORY
  June 20, 1910

[1] *Astrophysical Journal,* **29**, 233, 1909.

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